# Variance-of-variance risk premium

Article (Accepted Version)

# VARIANCE-OF-VARIANCE RISK PREMIUM

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*forthcoming in the Review of Finance*

**Abstract.** This paper explores the premium for bearing the variance risk of the VIX index, called the variance-of-variance risk premium. I find that during the sample period from 2006 until 2014 trading strategies exploiting the difference between the implied and realized variance of the VIX index yield average excess returns of -24.16% per month, with an alpha of -16.98% after adjusting for Fama-French and Carhart risk factors as well as accounting for variance risk (both highly significant). The paper provides further evidence of risk premium characteristics using corridor-variance swaps and compares empirical results with the predictions of reduced-form and structural benchmark models.

**Key words:** VIX, stochastic volatility-of-volatility, variance risk premium

**JEL:** C15, C32, G13, G15

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I would like to thank the editor (Bernard Dumas), and two anonymous referees for their very constructive comments that led to many improvements of the manuscript. Seminar participants at the University of Maastricht, University of Sussex, WHU and the University of Liverpool have provided many valuable suggestions. I am indebted to Johannes Rauch for many discussions on model-free risk premia. All remaining errors are my own responsibility.

The heteroskedasticity of equity index returns is one of the most prominent stylized facts in the finance literature. Empirical and theoretical studies over the last decades have investigated the effects of stochastic volatility on a wide range of financial applications such as derivative pricing or investment decisions (see for instance Bakshi *et al.*, 1997 or Liu and Pan, 2003). The random nature of variance also raises the question whether investors demand a premium for holding variance-sensitive assets. By comparing the prices of synthetic variance swaps with realized variances, Carr and Wu (2009) conclude that the market demands a significant premium for bearing the variance risk of S&P 500 index returns. Following this important finding, theoretical as well as empirical studies have contributed substantially to the understanding of higher-order risk, related premia and their wider economic implications (for instance Neuberger, 2012, Kozhan *et al.*, 2013, Martin, 2013 or Bondarenko, 2014).

As a consequence of the importance of stochastic volatility for market participants, the VIX index (published by CBOE[1]) has become a major benchmark in the finance industry as well as in academic research. The index can be interpreted as a measure of option-implied volatility of S&P 500 index returns and also serves as an approximation of 30-day variance swap rates (CBOE, 2009). The VIX is not a traded instrument, but to provide investors with direct access to volatility risk, futures and options on the VIX index have been successfully launched in 2004 and 2006, respectively. VIX options also provide investors with exposure to the volatility of the VIX process.[2] Empirical findings in Mencía and Sentana (2013) suggest that this *volatility-of-volatility* (henceforth vol-of-vol) is time-varying and an important risk factor in explaining the market prices of VIX options. Kaeck and Alexander (2013) model the variance of the VIX process directly and show that such feature is also important for explaining many time-series properties of the index. Baltussen *et al.* (2014) find that vol-of-vol calculated from implied volatility measures is a predictor of future stock returns.

[1]Chicago Board Options Exchange

[2]Since European option prices depend on the current volatility of the underlying and on its evolution until the maturity of the option, the volatility of the VIX is a crucial determinant of VIX option prices.

In this paper, I study the so-called variance-of-variance risk premium (VVP) which is defined as the difference between the (ex-ante) risk-neutral variance and the (ex-post) realized variance of the VIX index over a specified time horizon.[3] Such analysis is important for at least two reasons: first, it provides empirical evidence whether investors demand risk premia related to the variability of the VIX index, and such results may serve as an important reference for market participants exposed to vol-of-vol risk. Second, option pricing applications require an understanding of whether such a premium exists in the market. Recent research in this strand of the literature is based on the assumption that no such risk premia exist (see Mencía and Sentana, 2013). In calculating model-free risk premia, this paper follows recent theoretical developments in the definition of realized variance and builds on a framework that is free of jump and discretization biases (for a detailed discussion see Neuberger, 2012 and Bondarenko, 2014). Using option data from April 2006 until August 2014, I find that the difference between the implied and realized variance of the VIX is significant and investment strategies designed to exploit this yield an average monthly return of -24.16%. More than three quarters of this return cannot be explained by standard risk factors, culminating in a highly significant alpha of -17.78 to -16.98% per month, depending on the exact model specification.[4]

Following this main result, various aspects of the VVP are explored in more detail: What drives the VVP? Is there a term structure of variance-of-variance risk? How does the VVP compare to other VIX option strategies? What is the relationship between the variance risk premium (VP) and the VVP? To study the first question, I investigate the explanatory power of standard asset pricing risk factors and find that the market return exhibits the highest explanatory power (with a highly significant and negative effect). The momentum and size factors have no significant relationship with VVP returns, whereas the book-to-market factor can explain some of the return variation. After controlling for

[3] I use the terms variance-of-variance and vol-of-vol interchangeably, although the most accurate terminology in this context would be *variance-of-VIX*.

[4] I also demonstrate that the variance-of-variance swap rates with different maturities are driven by a low number of risk factors, similar to the term structure of interest rates or volatility term structures (see Fengler, 2005).

variance risk, market index returns become insignificant, whereas the regression alpha remains unaltered. I then follow ideas in Andersen and Bondarenko (2010) and dissect the realized variance into up-variance and down-variance measures. Empirical results indicate that monthly risk premia are statistically significant, independent of whether the variance was accumulated in an up or down corridor. Up-variance trades provide a lower monthly return of -30.73% whereas the down-variance contributes only -15.00%, both statistically significant. Alphas for these trades are also significant with values between -21.30 and -16.65%. Interestingly, the explanatory power of standard risk factors differs markedly, with an R-squared of only 6% during periods of stagnant volatility compared to almost 60% during periods of upward moving volatility.

To address the second question, I study the return of VVP investment strategies over different holding periods and find that the return of a monthly variance-of-variance contract is indistinguishable from the holding-period return of trades with a two-, three- or four-month horizon, all yielding holding period returns of less than -20%. Interestingly, standard equity risk factors show a stronger relation to longer-term investments, providing evidence that short-term variance-of-variance risk premia provide more market-independent sources of risk. To understand the contribution of the variance risk along the term structure, I study the return of option trading strategies that liquidate longer-term investments early, and hence are designed to depend on the realized variance of longer-term VIX futures prior to their maturity. While I do not find any significant alpha for such investment strategies, returns on these investments are measured with considerable noise which may have an adverse affect on the power of these tests.

How compatible are these empirical findings with the prediction of standard option pricing models? To address this questions, I show that the size and sign of the monthly premium can be generated in extensions of VIX option pricing models introduced in Mencía and Sentana (2013) and Bardgett *et al.* (2013). Reconciling variance and variance-of-variance risk premia in a single model requires the separation of volatility and vol-of-vol risk. I provide new evidence on model specifications that allow to model both empirical features simultaneously.

Finally, I compare VVP trades with two other sets of option strategies. First, I compare VVP to other simple VIX option trades such as selling out-of-the-money (OTM) options or at-the-money (ATM) straddles. While some of these have high absolute returns over the sample period, I find no evidence that any of these returns are significantly different from zero or exhibit significant alphas. This provides not only insights into the nature of variance risk but also shows that VVP contracts may be interesting trading strategies for VIX option investors. In addition, this paper compares the VVP with the variance risk premium implied in S&P 500 index options. VVP investments provide interesting return characteristics beyond those of the variance risk premium of S&P 500 index returns.

The remainder of the paper is structured as follows: Section 1 presents the methodological framework and Section 2 introduces the data set. The main empirical results are provided in Section 3, and model-based evidence of VVP is presented in Section 4. I conclude in Section 5.

# 1 METHODOLOGY

The variance risk premium is defined as the difference between the realized variance of a financial instrument and its (ex-ante) risk-neutral expectation. A significant difference between these two variance measures indicates that investors require a risk premium to hold variance-sensitive assets. A large body of literature examines such risk premia by employing high-frequency returns and an approximation of the risk-neutral characteristic that ignores jump risk (see Carr and Wu, 2009). Following Neuberger (2012) and Bondarenko (2014), the realized variance over a partition $\Pi = \{t = t_0 < \cdots < t_n = T\}$ of the interval from $t$ to $T$ is defined as

$$RV_{t,T}^{\Pi} = 2\sum_{i=1}^{n}\left(r_i^F - \log\left[1 + r_i^F\right]\right), \tag{1}$$

where $r_i^F = F_{t_i,T}/F_{t_{i-1},T} - 1$ is the simple return of a futures contract $F_{t,T}$ (with fixed maturity $T$) between two points in the partition. Neuberger (2012) shows that this non-standard definition can be regarded as a generalized variance measure and that its risk-neutral expectation can be calculated in a model-free way if $F_{t,T}$ follows a martingale. Therefore, using this definition ensures that results are not affected by jump or discretization biases.[5] I follow standard practice and work with daily returns, i.e. the points in the partition are comprised of trading days between $t$ and $T$. To simplify notation, $\tau \equiv T - t$.

The risk-neutral expectation of realized variance, as defined above, can be calculated from observed vanilla option prices. Following Bakshi and Madan (2000) and Bondarenko (2014), this expectation is given by

$$IV_{t,T} \equiv \mathbb{E}_t^{\mathbb{Q}}\left[RV_{t,T}^{\Pi}\right] = 2 \times \mathbb{E}_t^{\mathbb{Q}}\left[\frac{F_{T,T}}{F_{t,T}} - 1 - \log\left[\frac{F_{T,T}}{F_{t,T}}\right]\right] = \frac{2}{B(t,T)}\int_0^{\infty} k^{-2}M(k,t,T)\,dk, \tag{2}$$

where $\mathbb{E}_t^{\mathbb{Q}}[\cdot]$ is the time-$t$ risk-neutral expectation, $M(K,t,T)$ is the time-$t$ market price of an OTM option with strike $K$ and maturity date $T$ and $B(t,T)$ denotes the price of a zero bond maturing at time $T$ with a notional of one.[6] I interpolate between observed strikes using a simple cubic spline and extrapolate the observed implied volatility curves by setting values beyond the quoted strike range equal to the last observed implied volatility, a procedure that has become standard in the related literature. Conceptually similar to the literature, the integral is approximated using small step sizes in the strike dimension. However, rather then keeping the step size constant over the whole strike range (like a fraction of an index point), I apply a simple adaptive numerical integration

[5] Although the choice of partition might influence the size of the risk premium. The change in risk premium is, however, only due to the profits of a simple futures trading strategy and hence working with the daily returns is in line with standard practice.

[6] Put (call) options are OTM if $F_{t,T} > K$ ($F_{t,T} < K$).

routine which samples more frequently in a range where the integrand changes rapidly. This simple procedure alleviates the discretization bias that can otherwise arise in markets with pronounced skews (see Jiang and Tian, 2005).

To shed light on whether risk premia differ during alternative VIX index regimes, I also consider corridor variance swaps. This analysis follows ideas in Andersen and Bondarenko (2010) and relies on the computation of synthetic corridor variance swaps similar to Carr and Madan (1998). I define the implied corridor variance with down and up barriers, denoted $B_d$ and $B_u$, as

$$IV_{t,T}^{B_d,B_u} = \frac{2}{B(t,T)} \int_{B_d}^{B_u} k^{-2} M(k,t,T)\, dk. \tag{3}$$

The realized leg is given by

$$RV_{t,T}^{\Pi,B_d,B_u} = \sum_{i=1}^{n} g(F_{t_i,T}) - g(F_{t_{i-1},T}) - g'(F_{t_{i-1},T}) \times (F_{t_i,T} - F_{t_{i-1},T}), \tag{4}$$

with

$$g(x) = \begin{cases} 2 \times \left(-\log B_u - \frac{x}{B_u} + 1\right) & \text{if } x > B_u \\ -2\log x & \text{if } x \in [B_d, B_u] \\ 2 \times \left(-\log B_d - \frac{x}{B_d} + 1\right) & \text{if } x < B_d \end{cases}$$

and $g'(x)$ denotes the partial derivative of $g(x)$. This definition converges to the realized variance in Equation (1) for $B_d = 0$ and $B_u \to \infty$. The realized corridor variance is now determined by the level of the futures prices at the partition points, as well as the corridor barriers $B_u$ and $B_d$. First, if both futures prices are within the corridor, the contribution to the realized corridor variance coincides with that of a standard variance swap. Second, if the futures price jumps into the corridor from one partition point to the next, the contribution to realized variance is adjusted and only the distance from the barrier level to the new futures price enters the return calculation. Third, if the futures prices are below or above the corridor at two consecutive partition points, then the price movement does not enter the realized corridor variance. And forth, if the futures price jumps across the corridor between two partition points, then the return between the two barriers is adjusted by the distance of the final value from the near barrier.[7] The contract defined in Equation (4) is a special case of the *generalized variance* contract. Bondarenko (2014) shows that the definition of this contract retains the virtue of discretization and model invariance. Calculating investment returns for variance-of-variance swaps with different corridors allow us to dissect the pricing of variance-of-variance risk and to disentangle the types of risk that are priced in VIX options.

To specify the barrier levels over the sample period, I follow Andersen and Bondarenko (2010) and construct the ratio $R_{t,T}(K) \equiv P(K,t,T)/(P(K,t,T)+C(K,t,T))$ which can be calculated from market prices of VIX call and put options, denoted $C(K,t,T)$ and $P(K,t,T)$, respectively.[8] This function is monotonically increasing in the strike with $R_{t,T}(0) = 0$ and $R_{t,T}(\infty) = 1$ for all $t$ and $T$ and its inverse $K_q = R^{-1}_{t,T}(q)$ can be used to define the barriers of corridor variance swaps in a time-consistent way. For the main empirical results, I focus on breaking the variance into down- and up-variance measures which are labeled $\mathrm{VVP}_{0,50}$ (with $B_d = 0$ and $B_u = F_{t_0,T}$) and $\mathrm{VVP}_{50,100}$ (with $B_d = F_{t_0,T}$ and $B_u = \infty$). To check the robustness of these results I also break the range into three groups: up-variance ($\mathrm{VVP}_{67,100}$, for which $B_d = K_{0.67}$ and $B_u = \infty$), center-variance ($\mathrm{VVP}_{33,67}$, for which $B_d = K_{0.33}$ and $B_u = K_{0.67}$) and down-variance ($\mathrm{VVP}_{0,33}$, for which $B_d = 0$ and $B_u = K_{0.33}$).

[7] It is possible to express the realized leg of the corridor variance in a more compact notation as follows: $RV^{\Pi,B_d,B_u}_{t,T} = 2\sum_{i=1}^{n} \frac{F_{t_i,T}}{F^c_{t_i,T}}\left(\frac{F^c_{t_i,T}}{F^c_{t_{i-1},T}} - 1\right) - \log\frac{F^c_{t_i,T}}{F^c_{t_{i-1},T}}$, where $x^c := \max\left[B_d, \min\left[B_u, x\right]\right]$. I am very grateful to a referee for pointing me towards this contract definition.

[8] As before, $K$ denotes the strike price, $t$ is the current time, and $T$ is the maturity date.

## 2 DATA

VIX futures data are obtained from the CBOE website. VIX futures commenced trading on March 26, 2004 and the data cover daily price information until August 2014. Typically on each trading day during the sample between three and six different maturities are traded. Settlement prices are used for the calculation of realized statistics (defined above). Furthermore, VIX option data are obtained from Market Data Express from April 2006 until August 2014. VIX options also trade for several maturities on every trading day, typically with monthly expiries for upcoming months and a quarterly cycle thereafter. The maturity date of VIX futures and options is the Wednesday 30 days prior to the third Friday in the following month which is the S&P 500 index option (ticker: SPX) expiry date. This guarantees that, at expiry, the options used for calculation of the VIX index have exactly 30 days to maturity. I apply standard filters to the raw data to ensure that empirical results are not affected by illiquid quotes or obvious recording errors. In particular, I discard options that violate standard no-arbitrage relationships. After applying these filters, the database used in this study contains more than half a million VIX option quotes.[9]

For the comparison of variance risk premia and variance-of-variance risk premia, I also collect data on S&P 500 index options from OptionMetrics for the period from January 1996 until August 2014 (the longest sample period available at the time of writing). S&P 500 index options are European-style contracts and are among the most liquidly traded equity derivative instruments. On every trading day in the sample, I back out implied futures prices from put-call parity to circumvent the estimation of a dividend yield (using the ATM option pair for which the absolute price difference between put and call price is the smallest). I then apply, as before, a range of standard filters to remove quotes that fall outside standard arbitrage bounds (see Bakshi *et al.*, 1997). As a substitute for the

[9]Note that VIX options can be interpreted as options on VIX futures that have the same maturity as the options.

unobservable risk-free rate of interest I use interpolated rates from the OptionMetrics zerocurve file.

# 3 EMPIRICAL RESULTS

## *3.1 PRELIMINARY DATA ANALYSIS*

I use the term VVIX for normalized implied variance measures of the VIX which are defined as $\tau^{-1} \times IV_{t,T}$ (using VIX options for the calculation of the implied characteristic $IV_{t,T}$).[10] These measures can be constructed for different time horizons and this paper first focuses on 45, 90 and 135 days to maturity. To this end, I calculate VVIX for all available VIX option maturity dates and for each trading day in the sample. I then interpolate using a shape-preserving cubic interpolation method to construct time series with constant time to maturity.[11] I deviate from the standard 30 days to maturity as the shortest maturity to guarantee that no extrapolation is required and no options with less than one week to maturity are used in the index construction. Since long-term options can be fairly illiquid, I do not extent the analysis beyond 135 days.[12]

[Figure 1 about here.]

Figure 1 displays the evolution of the indices over the sample period. There are several noteworthy features. First (and unsurprisingly), the variance of the VIX is time-varying. Second, spikes in the short-term VVIX are observed relatively frequently. The events

[10]Note that the VVIX indices published by CBOE are measured in terms of volatility and therefore are defined as $\sqrt{\tau^{-1}\,\mathbb{E}_t^{\mathbb{Q}}\left[IV_{t,T}\right]}$. Since the focus of this paper is on variance contracts, I deviate slightly from the CBOE name convention.

[11]I initially used linear interpolation in VVIX indices and empirical results based on this method only differ marginally from the results presented below. The term structure of VVIX indices is typically downward sloping and convex, hence the chosen interpolation procedure reduces a small bias that would result from simple linear interpolation.

[12]It is, however, important to note that for the main empirical results of this paper, I use monthly returns on a 30 days-to-maturity variance of variance swap. This guarantees that for the variance-of-variance contract studied below, no interpolation in the time dimension is required.

around the Lehman default and the subsequent financial market crisis led to an increase in the volatility of the VIX index, but contrary to many other financial variables the spikes during this period are not exceptional. The most extreme vol-of-vol levels are in fact observed in 2007 and 2010. Third, the speed of mean reversion is stronger than for the VIX index itself and the first-order autocorrelation of the indices increases with their maturity. The short-term index exhibits a first-order autocorrelation of 0.9559 whereas the coefficient increases to 0.9805 for the 135-day index. And fourth, a downward sloping term structure of VVIX indices is observed throughout the whole sample period where the long-term VVIX index is consistently lower than the short-term index. This feature can be explained by the fact that the volatility of VIX futures increases for shorter-maturity futures contracts (see Alexander and Korovilas, 2013).

[Table 1 about here.]

Table 1 provides summary statistics for the three VVIX indices. Panel A confirms that the term structure of VVIX indices is downward sloping with mean, standard deviation and skewness monotonically decreasing for increasing index maturities. Panel B focuses on properties of (daily) first-order differences. It is evident that the index with the shortest time to maturity is the most volatile with a standard deviation several times the standard deviation of the 135-day index (0.05 compared to 0.01). All indices are positively skewed and show high levels of kurtosis. As a result, first-order differences in the indices are highly non-normal (which unreported tests confirm at the highest significance levels). Columns (6) to (9) of Table 1 provide results of a standard principal component analysis (PCA). These results indicate that the first principal component covers 91% of the variation in the multivariate system, whereas further 7% are attributed to the second principal component. These findings compare to other term structures, such as interest rates, implied volatilities or VIX indices. In Panel C, I normalize the first-order changes by their sample standard deviations, hence the PCA for these time series is based on the correlation rather than the covariance matrix. The interpretation of the components as shift, tilt and curvature is evident from these results. In addition, normalizing the time series leads to a stronger influence of the second and third component which now cover 19% of the variation. For completeness, Panel D reports the results for changes in the logarithm of the VVIX. Previous findings (including positive skewness and excess kurtosis) are confirmed with only minor quantitative differences.

### *3.2 RISK PREMIA AND THE PERFORMANCE OF VIX OPTION STRATEGIES*

[Figure 2 about here.]

I first explore the size and significance of the VVP over the sample period. The left graph in Figure 2 compares monthly realized variances of nearby VIX futures to their implied counterparts; the right graph provides the corresponding (excess) returns which are defined as $RV^{\Pi}_{t,T}/IV_{t,T} - 1$. Each month the return is calculated such that $T$ is the expiry date of VIX options in the next calendar month and $t$ is the trading day following the expiry date of the current month. Since VIX futures maturities coincide with these expiry dates this approach requires no interpolation of futures prices or VVIX indices at different maturities. Similar to standard variance swap investments, for most months the realized variance is below its implied characteristic, hence returns are often highly negative (see Bondarenko, 2014). Only occasionally, especially during excessively volatile market regimes, realized characteristics exceed their implied counterparts and large positive returns can be realized.

[Table 2 about here.]

Table 2 provides summary statistics for the VVP. The average monthly return for the VVP trade described above is -24.16%, and despite the relatively short sample period this return is highly statistically significant with a $t$-statistic of -3.39. I compare these returns to other popular option strategies commonly applied to equity index options (see Coval and Shumway, 2001 or Broadie *et al.*, 2009) and select buying OTM VIX options with fixed moneyness levels, buying the closest to the money option and an ATM VIX straddle. Average monthly returns for such strategies (for moneyness levels between 0.8 and 1.2) are also reported in Table 2.[13] Selling OTM call and put options can generate large absolute returns, however none of these are significantly different from zero with relatively low $t$-statistics throughout. A simple ATM straddle yields a return of roughly -4.45% per month, but only with a $t$-statistic of -0.44. In addition to a pure return analysis, I also report standard risk-adjusted performance measures for all strategies (Sharpe ratio, Sortino ratio, Stutzer index[14]). These risk-adjusted measures confirm that the VVP trade is the most successful strategy after adjusting for (potentially non-normal) risk.

One caveat of the alternative option strategies presented above is that they may be exposed to both price and variance risk (see Broadie *et al.*, 2009). ATM straddles held over a monthly period, for instance, are delta-neutral only at the inception of the trade. Since the underlying VIX index is very volatility (see Table 1), straddles may move in- or out-of the money during a trading month and their P&Ls may be affected by both price and volatility risk. To study whether this affects the conclusions, I calculate the returns of previously reported option strategies and delta hedge the exposure to the underlying VIX index. To do so, I calculate the Black delta of the option position (i.e. the sensitivity with respect to the VIX futures contract) on each trading day during the month and offset the delta risk by trading in VIX futures contracts. The returns for two delta-hedged option strategies are presented in Table 2. Delta-hedged straddle returns are now closer to the VVP return with a monthly average of -9.87% and a $t$-statistic of -1.85. The remaining difference between the performance of the two strategies may have several potential reasons. First, delta-hedged returns require a model assumption (the Black model in this case), and hence their returns may not be completely insensitive to underlying price movements. Second, although (approximately) delta-hedged, the

[13]Since VIX options are quoted for fixed strikes and not fixed moneyness levels, I use the same interpolation technique as for the calculation of the VVIX to generate constant moneyness prices each month.

[14]Refer to Bondarenko (2014) for more detail on these performance measures.

variance risk exposure of such strategies is time-varying. And third, a daily hedging strategy may not completely remove all underlying price risk if the underlying is very volatile (i.e. has high gamma). Buying ATM options and delta-hedging their VIX futures exposure leads to a return of -15.10% ($t$-statistic: -1.02) whereas the return of the un-hedged position is positive.[15]

[Figure 3 about here.]

These empirical results suggest that the synthetic variance-of-variance swap reliably captures the VVP and also compares well to competing option strategies. To understand whether the risk premium is regime specific, I extend the results to corridor variance swaps. Figure 3 provides up- and down-variances and corresponding investment returns (which are denoted $\mathrm{VVP}_{0,50}$ and $\mathrm{VVP}_{50,100}$) for the entire sample period. In general, both up- and down-variance trades are highly skewed and leptokurtic, with the up-trade providing more extreme returns than the down-trade. In particular, the realized up-variance measure is equal to zero for quite a few months in the sample, especially following a peak in the VVIX index. On the other hand, during few months the realized up-variance exceeds the implied characteristic several times, culminating in extreme positive returns for the strategy. The down-trade was highly lucrative to writers until 2011 and only after this occasional extreme positive outliers are observed.

[Table 3 about here.]

Table 3 reports average returns and several performance measures for different corridor variance trades over the sample period from 2006 until 2014. Overall, the up-trade $\mathrm{VVP}_{50,100}$ provides an average monthly return of -30.73% with a $t$-statistic of -2.58. The average return of the down-trade $\mathrm{VVP}_{0,50}$ is -15.00% per month, also significant with a $t$-statistic of -2.11. These findings suggest that during the sample the exposure to both

[15] I omit the return characteristic for other delta-hedge option strategies as they resemble closely the properties of the un-hedged options when further out-of-the money.

up- and down-variance is compensated and that VIX futures return variance in both corridors contributes to the overall premium. The return for an up-trade is considerably larger, nevertheless I find no statistical significance for return differences of $\mathrm{VVP}_{0,50}$ and $\mathrm{VVP}_{50,100}$. The results for the second partition of the distribution confirms the two main findings. First, variance exposure is priced in all corridors. And second, the highest average return is observed for trading up-variance with $\mathrm{VVP}_{67,100}$ exhibiting an average monthly return of -37.93% and a t-statistic of -2.41, whereas the return of the down-variance contract is merely -21.64% ($t$-statistic of -2.64). As before I find no statistical differences between average returns for different corridors.

### *3.3 ALPHAS AND RISK FACTOR SENSITIVITIES*

To test how much of the excess return is due to the correlation with well-known risk factors, I study the following, standard regression model:

$$r_{it} = \alpha_i + \beta_{m,i}\,(r_t^m - r_{ft}) + \beta_{smb,i}\,r_t^{smb} + \beta_{hml,i}\,r_t^{hml} + \beta_{umd,i}\,r_t^{umd} + \beta_{vixf,i}\,r_t^{vixf} + \varepsilon_{it} \quad (5)$$

where $r_{it}$ is the excess return (in month $t$) of the strategy under consideration (denoted $i$), $r^m$ is the market return, $r^{smb}$ denotes the return of the size portfolio (SMB), $r^{hml}$ denotes the return of the book-to-market portfolio (HML), $r^{umd}$ is the return of the momentum portfolio (UMD) and $r^{vixf}$ denotes the simple return of monthly VIX futures.[16] The choice of risk factors is partly motivated by standard asset pricing models, and partly by the importance to account for varriance risk that may be correlated with VVP. The risk-free rate $r_f$ as well as all other historical factor returns are downloaded from Kenneth French's website. I routinely adjust for heteroskedasticity and autocorrelation in the error terms $\varepsilon_{it}$.

[16] VIX futures returns are calculated such that they coincide with the maturity of the option strategy. For instance when studying the VVP over a month I use the futures with the same maturity as the options used in VVP.

[Table 4 about here.]

Table 4 reports the regression results for two sets of risk factors. First, I estimate model (5) with only the market return (thus setting $\beta_{smb,i} = \beta_{hml,i} = \beta_{umd,i} = \beta_{vixf,i} = 0$), and second with no restrictions on the beta coefficients. I find a significant negative relationship between VVP and the market index with a highly negative beta of -7.27 ($t$-statistic: -5.08). This finding is similar in magnitude to the variance risk premium regressions in Carr and Wu (2009), Kozhan *et al.* (2013) and Bondarenko (2014), who find market betas between -4.51 and -8.53 for their regressions of the variance risk premium return on market excess returns. The CAPM alpha of VVP is -17.78% and highly statistically significant with a $t$-statistic of -2.98. The $R^2$ of the regressions is 28.51%. The second regression which includes Fama-French and Carhart risk factors as well as VIX futures returns provides interesting differences. Most importantly, the stock market return becomes an insignificant determinant of VVP after controlling for variance risk (I confirm this by running additional regressions using all risk factors but $r^{vixf}$). This suggests that variance and variance-of-variance trading strategies have a similar dependence on stock market returns which cancels out if both variables are included in the regression.[17] SMB and UMD are un-related to VVP, the only other risk factor with a significant coefficient at the 5% level is HML with a $t$-statistic of -2.02. The alpha remains high after accounting for the additional risk factors with -16.98%, with a $t$-statistic of -3.28.[18]

Table 4 also reports alphas for the corridor variances trades, and there are some interesting findings. First, up-variance swap returns can be much better explained by

[17]Results presented further below provide further support for this finding.

[18]In an earlier version of this paper, I have also reported results from a Markov-switching regression that allows for two distinct regimes for the sensitivity of various risk factors. These additional results are available upon request. I have also run risk factor regressions for other VIX option related strategies (previously reported in Table 2); their results confirm earlier findings. Most importantly, selling OTM options, as well as the at-the-money straddle strategy do not provide significant alphas (all (absolute) $t$-statistics are below one). While these findings cannot be interpreted as general evidence in favor of VVP, they highlight the significant performance differences of VVP and simple VIX option portfolios. Detailed results are available upon request.

standard risk factors than the down-variance returns. This is reflected by the fact that the $R^2$s are substantially higher for the up-trade with levels between 59% and 60% (for the most general risk factor regressions), whereas standard risk factors explain less than 18% of the variance for the down-trade. A potential explanation for this finding is that risk-factors become more correlated during periods of market turmoil. This strong asymmetry can also be interpreted as evidence that the explanatory power of standard risk factors, and the market return in particular, are regime-dependent.[19] Second, the results for the center-variance indicate that standard risk factors are unsuccessful in explaining the return variation for center-corridor swaps. Also note that up- and down-corridor trades have alphas of similar size, suggesting that the additional return of the up-variance trade can be mainly attributed to the correlation with the market return. The only category for which alphas are only significant at a 10% level is the $\text{VVP}_{33,67}$ which exhibits values around -13%.

### *3.4 TERM STRUCTURE OF VARIANCE OF VARIANCE RISK PREMIA*

[Table 5 about here.]

Empirical results presented so far, as well as the empirical findings in the related literature, concentrate on one-month investment returns. Longer holding periods may provide additional insights into the pricing of vol-of-vol risk and this term structure of risk premia is the focal point of this section.[20] To provide preliminary results, I first construct return time series as follows: every month I select the trading day after the VIX option expiry and calculate variance-of-variance contract returns with two, three and four months to expiry. As for the monthly holding period, the investment dates are chosen in a way that no interpolation of VIX futures or VVIX indices is required, as

[19] As before, when including both market returns and VIX futures returns, the market beta becomes insignificant and the regime-specific behavior is picked up by the return of the VIX futures.

[20] As before, I do not use VIX options with more than 135 days to maturity and hence focus on a maximum holding period of four months.

contracts are held until expiration. Table 5 reports average returns and corresponding $t$-statistics of VVP trades with different holding periods.[21] The average risk premium over different holding periods is very stable and ranges from -20.44% for the two-month investment to -23.01% for the three month-investment. The premium remains highly significant for all investment horizons. For completeness, I also report the term structure of corridor-variances and find that for longer-term investment horizons, down-variance provides in-significant risk premia, while center-variance trades remain significant.

[Table 6 about here.]

Table 6 presents estimations of regression model (5) applied to the returns calculated over longer investment horizons.[22] These results confirm the findings in Table 4, in particular I find that VVP trades have significant alphas up to a four-month investment horizon. Alphas are only marginally lower than the alphas for one-month investments and range from -11.85% for the two-month strategy ($t$-statistic: -2.27) to -15.97% for the three-month returns ($t$-statistic: -4.03). Interestingly, the R-squared increases with the holding period and reaches 61.42% for four-month returns. This confirms that short-term, one-month trades have different investment characteristics and that longer-term investments are better explained by standard risk factors such as the market return of equities and/or the return of VIX futures. Other risk factors play a minor role in explaining long-term VVP investments, with SMB, HML and UMD all insignificant for longer-term horizons.

[Table 7 about here.]

To gain a better understanding of the term premia, I construct a second set of returns. I retain a monthly investment horizon, but enter every month a variance-of-variance

[21] By construction, these monthly returns are not independent over time and therefore the standard errors (and $t$-statistics) in Table 5 are adjusted for autocorrelation.

[22] For expositional clarity, I only report regression results for VVP. Results for corridor variance-of-variance swaps are available upon request.

contract with two (three or four) months to maturity, which is then sold one month later (and hence it is not held until its maturity). The return of such strategy only depends on the realized variance over the first month, as risks thereafter are fully hedged. Denoting the time of investment as $t_0$, the end of the holding period as $t_h$ and the maturity as $T$, the (excess) return of such strategy is defined as

$$\frac{1}{IV_{t_0,T}} \times \left(e^{-r_f \times (T-t_h)} \times RV^{\Pi}_{t_0,t_h} + IV_{t_h,T}\right) - 1. \tag{6}$$

I first provide empirical evidence of the average term premium in Table 7 and use the notation $\text{VVP}^{x \to y}$ for the premium of a contract initiated $x$ months before maturity and held for $y$ months. As shown in Table 7, these returns are substantially lower with values increasing from -4.55% for $\text{VVP}^{2\to1}$ to -0.70% for $\text{VVP}^{4\to1}$. Corresponding $t$-statistics suggest that the investment returns are not significantly different from zero with $t$-statistics between -0.29 ($\text{VVP}^{4\to1}$) and -1.14 ($\text{VVP}^{2\to1}$). Table 8 presents corresponding alphas and unsurprisingly, given earlier results, I find that none of the investment returns provide significant alpha values. Two further (untabulated) robustness checks highlight the high premium in the first month: (a) investment returns for longer holding periods, such as $\text{VVP}^{4\to2}$ or $\text{VVP}^{3\to2}$, also have insignificant alphas, and (b) alphas for long-short strategies consisting of a long contract in $\text{VVP}^{1\to1}$ and a short position in longer-dated contracts (such as $\text{VVP}^{4\to1}$, $\text{VVP}^{3\to1}$ or $\text{VVP}^{2\to1}$) have alphas (and Sharpe ratios) almost identical to the short-term investment highlighting the difference in the VIX variance risk at different horizons.

[Table 8 about here.]

Term-structure investment returns in Equation (6) depend on both the realized variance as well as the prevailing future spot rate $IV_{t_h,T}$. While term-structure strategies provide insignificant alphas, these results may be driven by the additional noise that results from the dependence on a future implied variance. To single out the contribution of $RV^{\Pi}_{t_0,t_h}$ one could use the forward implied variance to hedge the risk after $t_h$. This way the price of the hedge would be known at time $t$, and the return would not be calculated on the spot implied variance but only on a fraction that relates to the chosen investment horizon. Such strategy would reduce the noise in the investment returns as the only random return component is $RV^{\Pi}_{t,t_h}$. Unfortunately, it is not straight-forward to calculate forward rates in this set-up as only one option expiry for each underlying VIX futures contract is available, and no simple model-free relationship between VIX futures with different maturity exists. Therefore, the empirical findings, especially the insignificant term premia, have to be interpreted with the caveat in mind that this measure of term premia may be relatively noisy.

### *3.5 COMPARING VARIANCE OF VARIANCE RISK PREMIA WITH VARIANCE RISK PREMIA*

[Table 9 about here.]

Earlier results demonstrate that part of the VVP can be explained by the returns of VIX futures. The variance contract calculated from S&P 500 index options (VP) provides a second important benchmark as the construction of VVP and VP returns is conceptually similar. In this section I address two related issues. First I provide a simple comparison of the two risk premia over the sample period. I then use VP in the risk-factor regressions to provide a robustness check for whether previous results are sensitive to how variance returns are measured. To this end, I first construct VP returns for the S&P 500 index following the procedure outlined in Section 1. Average risk premia and performance statistics are reported in Table 9. Panel A summarizes the premia for the complete OptionMetrics sample from January 1996 until August 2014. I find an average risk premium of -20.77%, confirming results in Kozhan *et al.* (2013) whose sample lasts until January 2012. More interestingly, Panel B details the VP return for the same period that was used for VVP premium estimates. It is evident that over the shorter sample, the VP is reduced to -16.68% with a $t$-statistic of -2.03.

[Table 10 about here.]

For completeness, Table 10 shows the alphas and sensitivities with respect to the Fama-French risk factors. There are two noteworthy results. First, VP alphas over the 2006-2014 sample period are insignificant with $t$-statistics between -1.46 and -1.27, whereas VVP alphas over the same sample period are highly significant. Second, the variation of the VP explained by the market return and other Fama-French risk factors is almost twice as high as the variation explained in VVP. This indicates that VVP provides returns that exhibit more market-independent behavior.

[Table 11 about here.]

As a robustness check, I alter model specification (5) and include VP as an explanatory variable instead of $r^{vixf}$.[23] VP returns are more variable than VIX futures returns, and hence the beta coefficient for VP is lower than for $r^{vixf}$, albeit with very similar significance levels. VP returns are able to explain slightly more of the variation in VVP with an $R^2$ of 56% for the most general regression model. Overall, the earlier findings are confirmed by these additional results, in particular Table 11 shows that the alpha of VVP remains significant when including VP as an explanatory variable. Alphas are now -15.55% (CAPM regression, $t$ statistic: -3.07) and -15.53% (Fama-French-Carhart regression, $t$-statistic: -3.18), and hence similar to earlier reported values. I also follow Kozhan *et al.* (2013) and estimate the regression equation as part of a Seemingly Unrelated Regression (SUR), where the second regression equation is identical to the VP regression of this section, augmented by the VVP as an explanatory variable. For both

[23] For this analysis, a complication arises because VP and VVP are difficult to measure contemporaneously without relying on interpolation of market data because of the different expiry dates of VIX and SPX options. Depending on the month, expiry dates are usually either two or three trading days apart. I ignore this small difference and variance risk premia are calculated for each calendar month for the option maturity that expires closest to the VIX option maturity date.

the CAPM and the Fama-French-Carhart factors, the estimates of VVP alphas remain highly significant and this does not change my conclusions.[24]

### *3.6 TRANSACTION COSTS*

Empirical features that rely on mid prices of option quotes may not be exploitable after accounting for transaction costs. Driessen *et al.* (2009), for instance, show that the correlation risk premium calculated from option mid quotes is not robust to high equity option bid-ask spreads. Santa-Clara and Saretto (2009) investigate the effect of trading frictions on various S&P 500 index option strategies. This section discusses whether transaction costs prevent investors from exploiting the high VVP premium. Option market transaction costs are substantially higher than in equity markets. Broadie *et al.* (2009) report bid-ask spreads of the order between 3-10% depending on the moneyness of the contracts, for VIX options trading costs are even higher with average bid-ask spreads of the order of 20% of the mid price.

I follow closely Carr and Wu (2009). To exploit the profitability of the strategy, I assume that investors short the realized variance of the VIX index on a monthly basis by entering a VVP trade the day after the monthly VIX option expiry and holding the position until maturity (as in Section 3.2). To do so, I replicate the payoff of a VVP trade with time-$T$ cashflow equal to

$$P\&L_{t,T} = N(\$) \times \left[ IV_{t,T}^{\Pi,B_d,B_u} - RV_{t,T}^{\Pi,B_d,B_u} \right]. \tag{7}$$

where $N(\$)$ is the notional of the contract and $IV$ and $RV$ are defined above. Bondarenko (2014) shows that the realized leg of a (generalized) variance contract can be perfectly

[24] I also account for possible endogeneity of VVP and VP returns by using instrumental variables as in Kozhan *et al.* (2013) but also find that this is not changing any conclusions regarding the significance of the intercept. I follow Kozhan *et al.* (2013) and construct instruments from risk-neutral moments implied by S&P 500 and VIX options following the procedure in Bakshi *et al.* (2003).

replicated as follows:

$$RV_{t,T}^{\Pi,B_d,B_u} = \int_{B_d}^{B_u} g''(k)M(k,t,T)\,dk - \sum_{i=1}^{n} [g'(F_{t_i,T}) - g'(F_{t_0,T})] \times (F_{t_i,T} - F_{t_{i-1},T}) \quad (8)$$

where $g$ is defined in Section 1. Entering a short position in RV therefore requires a short position in VIX options at the forward cost $IV_{t,T}^{\Pi,B_d,B_u}$. I take into account the cost of the replication by calculating the option integral using bid prices and hence $IV_{t,T}^{\Pi,B_d,B_u}$ (and the profitability) is lowered compared to the the assumption of a mid-quote trade.[25] I provide average P&Ls for a range of different strategies: (a) with constant notional of $N(\$) = 100$ or $N(\$) = 100 \times \left[IV_{t,T}^{\Pi,B_d,B_u}\right]^{-1}$, (b) for standard VVP trades with $B_d = 0$ and $B_u = \infty$ and corridor variance swaps that use only the available range of options at the initiation of the contract (that is $B_d$ corresponds to the lowest strike available whereas $B_u$ corresponds to the highest strike available), and (c) I use partitions of one day or one week.

Table 12 reports the average monthly P&L for various strategies over the sample period, with and without adjusting for transaction cost. Overall, I find that all strategies provide a positive performance despite the substantial transaction cost in the VIX option market. The performance of the absolute strategy with $B_d = 0$ and $B_u = \infty$ and a realized variance based on daily returns provides a positive profit with a $t$-statistic of 2.32. Most strategies (especially for the weekly realized variance measure) are statistically significant at a 1% level, and all strategies remain significant at the 10% level. Results are also consistent with the intuition that the profitability of a strategy with $N(\$) = 100 \times \left[IV_{t,T}^{\Pi,B_d,B_u}\right]^{-1}$ should be lowered by approximately half of the average 20% bid-ask spread as this is the difference in the cost compared to trading at mid-prices (24.16% vs. 14.01%).

[Table 12 about here.]

[25] I follow Kozhan *et al.* (2013) by assuming zero trading costs in the underlying, as the cost tend to be marginal compared to the high bid-ask spreads in the option market.

While it is common in the related literature to rely on bid/ask prices provided by options exchanges, there is evidence in the literature that such approach is likely to be very conservative. Carr and Wu (2009) point out that bid-ask spreads in the broker dealer market for variance swaps on the S&P 500 index are much lower than the spread calculated from bid-ask spreads of quoted S&P 500 index options. If option bid and ask prices lead to substantially different bid-ask swap rates, this indicates either a significant underestimation of the hedging costs by broker dealers or that the hedging cost may be somewhat lower than what is implied in option quotes. In a recent paper, Muravyev and Pearson (2015) find support for the latter. They argue that by timing option trades the effective bid-ask spread may be substantially reduced, and conclude that *"the quoted spread overstates the cost of taking liquidity by a factor of almost two."* (p.29) It is therefore possible that VIX options investors may reduce transaction costs compared to the assumptions used in this paper.

# 4 VARIANCE-OF-VARIANCE PREMIUM IN BENCHMARK MODELS

This section provides simulation-based evidence. My aim is to investigate whether option pricing models employed in the literature are able to explain the size of the risk premium, as well as other facets of the data presented in previous sections.

## *4.1 STRUCTURAL BENCHMARK MODELS*

As the main benchmark, I employ an extension of the two-factor stochastic variance model proposed in Duffie *et al.* (2000), Egloff *et al.* (2010) and Bates (2012). The simulation results in this section rely on estimated parameters from Bardgett *et al.* (2013), and therefore I follow closely their specification of asset price dynamics. The model is labelled

SVSMRJ (stochastic volatility, stochastic mean reversion, jumps) and it is assumed that under the risk-neutral pricing measure $\mathbb{Q}$ the S&P 500 index $S_t$ evolves according to the following stochastic differential equations:

$$
\begin{aligned}
\frac{dS_t}{S_t} &= \left(r - q - \lambda_t^{sv}\psi^{\mathbb{Q}}\right) dt + \sqrt{v_t}\, dW_t^{s,\mathbb{Q}} + \left(e^{\xi_t^{s,\mathbb{Q}}} - 1\right) dN_t^{sv,\mathbb{Q}}, & (9)\\
dv_t &= \kappa_v^{\mathbb{Q}} \left(m_t - v_t\right) dt + \sigma_v \sqrt{v_t} \left(\rho\, dW_t^{s,\mathbb{Q}} + \sqrt{1-\rho^2}\, dW_t^{v,\mathbb{Q}}\right) + \xi_t^{v,\mathbb{Q}}\, dN_t^{sv,\mathbb{Q}}, & (10)\\
dm_t &= \kappa_m^{\mathbb{Q}} \left(\theta_m^{\mathbb{Q}} - m_t\right) dt + \sigma_m \sqrt{m_t}\, dW_t^{m,\mathbb{Q}} + \xi_t^{m,\mathbb{Q}}\, dN_t^{m,\mathbb{Q}}, & (11)
\end{aligned}
$$

where $r$ denotes the risk-free rate, $q$ the dividend yield and $\psi^{\mathbb{Q}} = E^{\mathbb{Q}}\left[e^{\xi_t^{s,\mathbb{Q}}}\right] - 1$ is the jump compensator; $v_t$ is the stochastic variance process with stochastic long-run variance $m_t$. The three Brownian motion processes $W^{s,\mathbb{Q}}$, $W^{v,\mathbb{Q}}$ and $W^{m,\mathbb{Q}}$ are independent. Jumps in the price and variance process occur simultaneously and at random times whenever the increments in the Poisson counting process are equal to one, i.e. $dN_t^{sv,\mathbb{Q}} = 1$. $N_t^{sv,\mathbb{Q}}$ is assumed to have state-dependent intensity $\lambda_t^{sv} = \lambda_c^{sv} + \lambda_v^{sv} v_t + \lambda_m^{sv} m_t$. The jump $\xi_t^{s,\mathbb{Q}}$ is normally distributed with mean $\mu_s^{\mathbb{Q}}$ and standard deviation $\sigma_s^{\mathbb{Q}}$, variance jumps $\xi_t^{v,\mathbb{Q}}$ are exponentially distributed with mean $\mu_v^{\mathbb{Q}}$. Jumps in the long-term variance $\xi_t^{m,\mathbb{Q}}$ occur with intensity $\lambda_t^m = \lambda_c^m + \lambda_m^m m_t$ and are also exponentially distributed with mean $\mu_m^{\mathbb{Q}}$. All jump distributions are independent of each other.

To simulate the realized statistics, assumptions on the measure change between the risk-neutral pricing measure $\mathbb{Q}$ and the statistical measure, denoted $\mathbb{P}$, are required. In line with Bardgett *et al.* (2013), I assume that under the measure $\mathbb{P}$ the dynamics are given by

$$
\begin{aligned}
\frac{dS_t}{S_t} &= \left(r - q + \eta_s v_t - \lambda_t^{sv}\psi^{\mathbb{P}}\right) dt + \sqrt{v_t}\, dW_t^{s,\mathbb{P}} + \left(e^{\xi_t^{s,\mathbb{P}}} - 1\right) dN_t^{sv,\mathbb{P}}, & (12)\\
dv_t &= \kappa_v^{\mathbb{P}} \left(\frac{\kappa_v^{\mathbb{Q}}}{\kappa_v^{\mathbb{P}}}\, m_t - v_t\right) dt + \sigma_v \sqrt{v_t} \left(\rho\, dW_t^{s,\mathbb{P}} + \sqrt{1-\rho^2}\, dW_t^{v,\mathbb{P}}\right) + \xi_t^{v,\mathbb{P}}\, dN_t^{sv,\mathbb{P}} & (13)\\
dm_t &= \kappa_m^{\mathbb{P}} \left(\theta_m^{\mathbb{P}} - m_t\right) dt + \sigma_m \sqrt{m_t}\, dW_t^{m,\mathbb{P}} + \xi_t^{m,\mathbb{P}}\, dN_t^{m,\mathbb{P}}. & (14)
\end{aligned}
$$

The dynamics results from standard assumptions on risk premia and allow the parameters of the jump sizes to differ across measures, as well as introduces diffusive premia that alter the drift term of the three processes.[26] It can be shown that the squared VIX index in this model is a linear function of the latent state variables $v_t$ and $m_t$. Using this standard result, derivatives on the S&P 500 index and the VIX index can be priced in a consistent way. This in turn allows us to simulate model-based predictions of the sign and size of VP and VVP contract returns.

Bardgett *et al.* (2013) calibrate the full model as well as nested specifications to VIX and S&P 500 index derivatives from March 2006 until October 2008. I use their calibrated parameter values (from their Dataset 4) for the two-factor variance model. A range of nested specifications have also attracted considerable attention in the literature. First, I use a two-factor variance model without jumps, labeled SVSMR. Second, the stochastic volatility and contemporaneous jump (SVCJ) model of Eraker *et al.* (2003) arises as a special case when the long-run volatility level and jump intensities are assumed constant, the stochastic volatility and jump (SVJ) model of Bates (1996) when jumps are further restricted to the underlying process and the stochastic volatility (SV) model of Heston (1993) when a pure diffusion model is assumed. To assess model risk and different data periods, I use parameter values from Eraker (2004) for the simulation of one-factor variance specifications (estimated to a dataset from 1987 until 1990).

[26]More precisely, I assume standard affine risk premia such that the structural form of the processes under both measures remains unaltered. In particular the long-term variance premium is given by $\kappa_m^{\mathbb{P}} - \kappa_m^{\mathbb{Q}}$ (with the restriction $\kappa_m^{\mathbb{Q}}\theta_m^{\mathbb{Q}} = \kappa_m^{\mathbb{P}}\theta_m^{\mathbb{P}}$). The (short-term) variance premium is given by $\kappa_v^{\mathbb{P}} - \kappa_v^{\mathbb{Q}}$. The jump risk premia are specified by the difference between the jump distribution parameters under the two measures. The jump distribution $\xi_t^{s,\mathbb{P}}$ is now normally distributed with mean $\mu_s^{\mathbb{P}}$ and standard deviation $\sigma_s^{\mathbb{P}}$, variance jumps $\xi_t^{v,\mathbb{P}}$ and $\xi_t^{m,\mathbb{P}}$ are exponentially distributed with mean $\mu_v^{\mathbb{P}}$ and $\mu_m^{\mathbb{P}}$, respectively. Jump intensities are assumed to remain unaltered. Finally, the equity risk premium is $\gamma_t = \eta_s v_t + \lambda_t^{sv}\left(\psi^{\mathbb{P}} - \psi^{\mathbb{Q}}\right)$.

### 4.2 REDUCED-FORM BENCHMARK MODELS

An alternative modeling approach is to specify the dynamics of the VIX index directly, such as in Mencía and Sentana (2013). The advantage of such (reduced-form) approach is that more realistic dynamics for the VIX may be assumed, however at the cost of not being able to link the S&P 500 and VIX derivative markets. I first follow Mencía and Sentana (2013) and model the log of the VIX index $V_t$ under the risk-neutral measure as

$$dV_t = \kappa(\theta_t - V_t)dt + \sqrt{\omega_t}\, dW_t^{V,\mathbb{Q}} \tag{15}$$

$$d\theta_t = \kappa_\theta(\theta_\theta - \theta_t)dt + \sigma_\theta\, dW_t^{\theta,\mathbb{Q}} \tag{16}$$

$$d\omega_t = -\lambda dt + dZ_t^{\theta,\mathbb{Q}}, \tag{17}$$

where $W^{V,\mathbb{Q}}$ and $W^{\theta,\mathbb{Q}}$ are independent Brownian motions and $Z^{\theta,\mathbb{Q}}$ follows a Lévy OU-$\Gamma$ process.[27] I label the model CTOUSV. Using the assumption that the transition from the risk-neutral measure to the real-world measure is specified as $dW_t^{V,\mathbb{Q}} = dW_t^{V,\mathbb{P}} + \eta_V \sqrt{\omega_t} dt$ and $dW_t^{\theta,\mathbb{Q}} = dW_t^{\theta,\mathbb{P}} + \eta_\theta dt$, Mencía and Sentana (2013) find that this model outperforms a number of alternative specifications, such as simpler two-factor models or models for the VIX index rather then its log, for the pricing of VIX options and futures.[28]

### 4.3 SIMULATION

All models introduced above are used to simulate variance-of-variance risk premia following the empirical setting in Section 3 as closely as possible. For the structural models of Section 4.1 I proceed as follows: (a) using an Euler discretization of the stochastic processes with 100 steps per day, eight years of daily values for the stochastic system (12) to (14) are simulated; (b) for every month (assuming 21 trading days) in the eight-year sam-

[27] More specifically, $Z^{\theta,\mathbb{Q}}$ is a jump process with intensity $\lambda$ and exponential upward jumps only.

[28] They estimate the model on two samples, from February 2006 until August 2008 and from February 2006 until December 2010.

ple, I calculate the realized statistic as in Equation (1).[29] For the implied statistic, I use standard Fourier inversion formulae to calculate VIX options for moneyness levels from 0.4 to 1.6 (in steps of 0.05) and follow the procedure outlined earlier to interpolate and extrapolate the resulting implied volatilities.[30] And (c), I calculate variance-of-variance risk premia for different maturities of one to four months. For the reduced-form models, I follow the same procedure by simulating the risk premium-adjusted version of (15) to (17).

[Table 13 about here.]

Table 13 provides simulation results for the VVP in structural models. The simulations reveal that model-based VVP risk premia are significantly different from the risk premium observed during the sample period. The one-factor structural pricing models predict a positive VVP between 2.09 and 5.56% per month and the 1% percentile for the model with the largest variations (SVCJ) is only -15% and therefore only a fraction of the empirical premium during the sample period. The two-factor structural pricing models also show relatively strong deviations from the empirical risk premium. I conclude that structural models nested in Equations (9) - (11) struggle to explain the size and the sign of the VVP. For comparison, in Panel B of Table 13 I report simulation results for VP. Except for SVSMR, all models produce on average risk premia that are similar to the sample mean of -16.68% and the structural models therefore only fail to produce variance premia in the VIX and not in the S&P 500 option market.[31]

The reduced-form model of Mencía and Sentana (2013) produces similar predictions, with average VVP returns of 2.73 and 3.29%, depending on the parameters of the processes (see Table 14). The standard deviation of the risk premium, however, is larger

[29] I assume a risk-free rate of 2% p.a. and zero dividend yield for the calculation of option and futures prices.

[30] This procedure has the advantage that all quantities are constructed in the same way as for the empirical analysis in Section 3 and can be directly compared.

[31] I have also simulated risk premia using other model parameters to understand whether the results are sensitive to these assumptions. While some parameters affect the simulated risk premia (as theoretically expected), simulation results for other parameter values remain at odds with the empirical findings and my results are not driven by the specific choice of parameters.

than in structural models and hence the empirical -24% return falls between the 1% and 5% percentile. The models also fail to explain longer-term variance risk premia based on overlapping four-month risk premium trades (untabulated).

### *4.4 MODEL EXTENSIONS*

Both model classes can be extended to accommodate more realistic variance-of-variance risk premia. For simplicity and tractability, Mencía and Sentana (2013) focus on a simple jump process for the variance of the VIX index and explicitly impose a no-risk-premium assumption by setting risk-neutral parameters equal to real-world parameters in the dynamics of $\omega$. In order to investigate the effect of risk premia in this process on the VVP, I replace Equation (17) by

$$d\omega_t = \kappa_\omega^{\mathbb{Q}} \left( \theta_\omega^{\mathbb{Q}} - \omega_t \right) dt + \sigma_\omega \sqrt{\omega_t}\, dW_t^{\omega,\mathbb{Q}} \tag{18}$$

and impose a risk premium by assuming that $dW_t^{\omega,\mathbb{Q}} = dW_t^{\omega,\mathbb{P}} + \eta_\omega \sqrt{\omega_t} dt$. Table 14 provides simulation results for a model with negative and zero $\eta_\omega = 0$ (labeled M1 and M2). These results demonstrate that such model extension can reduce the standard error of estimated VVP returns and the models yield realistic results in terms of the size of the premium with a mean of -28.54%. Although the results are based on hypothetical parameter values, they provide important insights into the necessary features for a reduced-form VIX option pricing model. The disadvantage of these modeling assumptions is that the characteristic function of such model extension is not available in closed form and needs to be calculated numerically.

[Table 14 about here.]

In the structural framework, the most crucial extension regards the modeling of vol-of-vol risk.[32] I assume that under the risk-neutral measure the dynamics of the stock

[32] I would like to thank one of the referees for suggesting this approach.

index are given by

$$\frac{dS_t}{S_t} = \left(r - q - \lambda_t \psi^{\mathbb{Q}}\right) dt + \sqrt{v_t}\, dW_t^{s,\mathbb{Q}} + \left(e^{\xi_t^{s,\mathbb{Q}}} - 1\right) dN_t^{\lambda,\mathbb{Q}}, \tag{19}$$

$$dv_t = \kappa_v^{\mathbb{Q}} \left(m_t - v_t\right) dt + \sigma_v \sqrt{v_t} \left(\rho\, dW_t^{s,\mathbb{Q}} + \sqrt{1-\rho^2}\, dW_t^{v,\mathbb{Q}}\right) + \xi_t^{v,\mathbb{Q}}\, dN_t^{\lambda,\mathbb{Q}}, \tag{20}$$

$$dm_t = \kappa_m^{\mathbb{Q}} \left(\theta_m^{\mathbb{Q}} - m_t\right) dt + \sigma_m \sqrt{m_t}\, dW_t^{m,\mathbb{Q}} \tag{21}$$

$$d\lambda_t = \kappa_l^{\mathbb{Q}} \left(\theta_l^{\mathbb{Q}} - \lambda_t\right) dt + \sigma_l \sqrt{\lambda_t}\, dW_t^{l,\mathbb{Q}} + \xi_t^{l,\mathbb{Q}}\, dN_t^{\lambda,\mathbb{Q}}, \tag{22}$$

$\lambda_t$ is the stochastic intensity of the Poisson process $N_t^{\lambda,\mathbb{Q}}$, $\xi_t^{l,\mathbb{Q}}$ is exponentially distributed (with mean $\mu_l^{\mathbb{Q}}$) and the Brownian motion $W_t^{l,\mathbb{Q}}$ is independent of $W_t^{s,\mathbb{Q}}$, $W_t^{v,\mathbb{Q}}$ and $W_t^{m,\mathbb{Q}}$. These modeling assumptions imply self-exciting behavior: a jump in the Poisson process $N^{\lambda,\mathbb{Q}}$ raises the likelihood of further jumps, thereby creating periods of increased jump activity. Self-exciting jump models have recently been studied in Fulop *et al.* (2014), Aït-Sahalia *et al.* (2015), Carr and Wu (2016) or Bates (2016). While the focus of previous studies is on modeling the return process, my main motivation for using stochastic jump intensities is to generate more realistic stochastic vol-of-vol dynamics. Under the model assumptions, Iit follows that

$$Var\left(dv_t\right)/dt = \sigma_v^2 v_t + \lambda_t E_t^{\mathbb{Q}}\left[\left(\xi_t^{l,\mathbb{Q}}\right)^2\right] \tag{23}$$

and therefore variance-of-variance risk exhibits uncertainty modeled by $\lambda_t$ in addition to the usual variance level effect.[33]

I repeat the simulation exercise using the stochastic intensity model in Equations (19) - (22) to study whether the model may capture both negative VP and VVP returns. To focus on the main ideas, I use parameter values that are broadly in line with the findings in the related literature. I first use $\kappa_v^{\mathbb{Q}} = 4$, $\kappa_v^{\mathbb{P}} = 2$, $\sigma_v = 0.35$, $\rho = -0.6$, $\kappa_l^{\mathbb{Q}} = 5$, $\theta_l^{\mathbb{Q}} = 10$, $\sigma_l = 0$ and $\mu_l^{\mathbb{Q}} = 2$. Note that I assume a lower value for $\sigma_v$ as variance is now subject

[33] One may also model $\lambda_t$ as a pure CIR process, but self-exciting processes have the advantage that $v$ and $\lambda$ are both affected by increments in the same Poisson process and hence one can use these jumps to model correlation of the variance and its variance-of-variance.

to more frequent jumps (around 17 per year on average). I assume, in line with Eraker (2004), that risk-neutral variance jumps are on average higher than under the real-world measure: $\mu_v^{\mathbb{Q}} = 0.013$ and $\mu_v^{\mathbb{P}} = 0.005$. The most highly parameterized model further assumes $\kappa_m^{\mathbb{Q}} = 1$, $\theta_m^{\mathbb{Q}} = 0.15^2$ and $\sigma_m = 0.10$, and that $-\xi_t^{s,\mathbb{Q}}$ is exponentially distributed with mean $\mu_s^{\mathbb{Q}} = 0.01$ and $\mu_s^{\mathbb{P}} = 0.005$, respectively. In the simulation exercise, I assume identical real-world and risk-neutral parameters for $m_t$ and $\lambda_t$, Fulop *et al.* (2014) discuss measure changes in self-exciting jump specifications. I have also experimented with a range of alternative return jump distributions but find that these are of second-order importance.

Column *Model 1* of Table 15 provides simulation results for VP and VVP returns for the specification described in the previous paragraph. The simulation results suggest that the direct modeling of stochastic vol-of-vol leads to realistic risk premium returns of -28.93% (VVP) and -26.35% (VP), in line with empirical evidence in Section 3. Most importantly, the extended model specification allows to generate both negative VP and VVP returns. To gage the effect of the different components of model (19) - (22), in column *Model 2*, I impose a constant mean-reversion level $m_t = 0.15^2$ by setting $\sigma_m = 0$ and observe – in line with previous results – that this has no major effect on the results. In column *Model 3*, I set $\mu_v^{\mathbb{Q}} = \mu_v^{\mathbb{P}} = 0.005$ and observe that a high jump size average under $\mathbb{Q}$ introduces a wedge between risk-neutral and real-world dynamics that drives VVP. In column *Model 4*, I replace the price jump assumptions of *Model 1* by $\mu_s^{\mathbb{Q}} = \mu_s^{\mathbb{P}} = 0$ and find that while this has a marked effect on the variance premium contract, VVP returns remain similar to models with price jumps.

Overall, the simulation results suggest that in order to model both VIX and SPX options simultaneously, a stochastic vol-of-vol process is of first-order importance (compared to price jumps or stochastic mean reversion). To estimate such model specifications and to test their empirical pricing and hedging performance is an interesting avenue for future research.

[Table 15 about here.]

# 5 CONCLUSION

This paper studies risk premia for bearing the variance risk of the VIX index termed the variance-of-variance premium (VVP). I find that investors demand a significant premium of approximately -24% per month for bearing this risk. These results may be interesting for several reasons: first, investment strategies based on the variance-of-variance risk premium offer attractive returns and compare favorably to the returns of very popular variance-swap trades. And, second, the empirical results highlight the importance of taking stochastic volatility-of-volatility effects into account when pricing options on the S&P 500 and VIX index in a structural model framework.

# A APPENDIX

## A.1 FURTHER SIMULATION RESULTS

An alternative modeling approach is to consider the effect of non-affine model dynamics, such as CEV extensions of the SV model. Using Ito's Lemma and the fact that the squared VIX is a linear function of variance, i.e. $\text{VIX}_t^2 = a + b_v v_t$, I obtain

$$Var\left(\frac{d\text{VIX}_t}{\text{VIX}_t}\right)/dt = \frac{b_v^2 \sigma_v^2 v_t}{4\left(a + b_v v_t\right)^2}. \tag{24}$$

Therefore, assuming realistic parameter values, the variance of VIX index returns and the variance of the S&P 500 are inversely related for almost the entire range of $v_t$, a fact that relates to the property that the diffusion part of stochastic volatility in the Heston model is independent of $v_t$ (see also Heston (1993) for a discussion). Does this model feature contribute to the unrealistic VVP returns in Table 13? As the empirical results in the body of the paper are based on VIX futures rather than the spot VIX, I calculate the realized variance using S&P 500 index returns and the realized variance of the VIX index for every month in the sample and find that the two variables have indeed a positive (highly significant) correlation of 45.56%. Standard affine models fail to explain this empirical finding.

Non-affine models may potentially alleviate the shortcomings discussed in the previous paragraph. Non-affine models have been advocated by a large number of empirical studies, such as Jones (2003), Christoffersen *et al.* (2010) or Kaeck and Alexander (2012). Despite their empirical success, non-affine models are used less frequently in the literature as they lack closed form solutions of their characteristic function and hence standard Fourier pricing algorithms are not applicable. Nevertheless, the theoretical value of the squared VIX index remains linear in variance and hence I can compare the implications of non-affine model dynamics on the correlation between the realized variance of the VIX and the S&P 500 index. To provide some intuition, I use estimated model parameters from Eraker (2004), and simulate eight years of monthly realized spot variances and realized variances of the VIX index. The left graph in Figure 4 confirms the theoretical findings above, that is an inverse relationship between the two realized variance measures (average correlation - 51.43%).[34] I then repeat the simulation replacing the affine CIR model with the CEV (constant elasticity of variance) model

$$dv_t \;=\; \kappa_v\left(\theta_v - v_t\right)\,dt + \sigma_v {v_t}^{\gamma}\left(\rho\,dW_t^s + \sqrt{1-\rho^2}\,dW_t^v\right). \tag{25}$$

To this end, I match $\sigma_v$ in the CEV model such that the average diffusion term is identical to SV. The right graph in Figure 4 demonstrates that a model with $\gamma = 1$ may provide model features in line with the empirical correlation observed over the sample period. A potentially fruitful avenue for future research is therefore to develop efficient pricing algorithms that allow for non-affine model dynamics, Sepp (2015) is a recent development in this area of research.

[Figure 4 about here.]

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[34] I use 100,000 simulation runs for these graphs.

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**Table 1.** Summary statistics and principal component analysis.

This table reports the summary statistics and results from a principal component analysis (PCA) of VVIX indices with three maturities (45, 90 and 135 days) over the sample period from April 2006 until August 2014. Column (2) to (5) report the mean, standard deviation (*std*), skewness (*skew*) and kurtosis and columns (6) to (8) report the eigenvectors of the three principal components. Column (9) (%) summarizes the amount of variation explained by the first (first line), second (second line) and third (third line) principal component. Results in Panel A are based on levels, Panel B on daily first differences, in Panel C these differences are normalized by their sample standard deviation. Panel D is based on first differences of the log of the index values.

| Maturity | mean | std | skew | kurtosis | PC(1) | PC(2) | PC(3) | (%) |
|---|---|---|---|---|---|---|---|---|
| Panel A: Levels | | | | | | | | |
| 45 days | 0.66 | 0.18 | 0.87 | 3.98 | 0.82 | 0.56 | -0.11 | 0.95 |
| 90 days | 0.47 | 0.11 | 0.38 | 2.69 | 0.48 | -0.57 | 0.67 | 0.05 |
| 135 days | 0.37 | 0.07 | 0.36 | 2.91 | 0.31 | -0.61 | -0.73 | 0.00 |
| Panel B: Changes | | | | | | | | |
| 45 days | 0.00 | 0.05 | 1.09 | 12.74 | -0.93 | -0.37 | -0.00 | 0.91 |
| 90 days | 0.00 | 0.02 | 0.62 | 11.22 | -0.33 | 0.82 | -0.48 | 0.07 |
| 135 days | 0.00 | 0.01 | 0.99 | 15.26 | -0.18 | 0.44 | 0.88 | 0.02 |
| Panel C: Normalized Changes | | | | | | | | |
| 45 days | 0.00 | 1.00 | 1.09 | 12.74 | 0.57 | 0.68 | 0.46 | 0.81 |
| 90 days | 0.00 | 1.00 | 0.62 | 11.22 | 0.59 | 0.05 | -0.80 | 0.12 |
| 135 days | 0.01 | 1.00 | 0.99 | 15.26 | 0.57 | -0.73 | 0.38 | 0.07 |
| Panel D: Log Changes | | | | | | | | |
| 45 days | 0.00 | 0.07 | 0.97 | 8.70 | 0.83 | 0.55 | -0.09 | 0.83 |
| 90 days | 0.00 | 0.05 | 0.49 | 6.97 | 0.46 | -0.58 | 0.67 | 0.12 |
| 135 days | 0.00 | 0.04 | 0.65 | 10.77 | 0.32 | -0.60 | -0.73 | 0.05 |

**Table 2.** Variance-of-variance risk premium and VIX option returns.

This table reports the statistics for the variance-of-variance risk premium (VVP), other VIX option strategies and the market return over the sample period from April 2006 until August 2014. VVP is measured as the monthly return defined by $RV^{\Pi}_{t,T}/IV_{t,T} - 1$, where $IV_{t,T}$ is the implied variance and $RV^{\Pi}_{t,T}$ the realized daily variance over a monthly period. Columns (4) to (6) report the Sharpe ratio, the Sortino ratio and the Stutzer index. For comparison, the table also reports the return statistics of simple VIX option strategies such as buying out-of-the-money (OTM) options with different moneyness levels and an at-the-money (ATM) straddle. Delta-hedged option strategies are labeld *DH*.

| Strategy | mean return | ($t$-stat) | Sharpe | Sortino Ratio | Stutzer |
|---|---|---|---|---|---|
| VVP | -24.16 | (-3.39) | -0.34 | -0.48 | -0.31 |
| Market | 0.88 | (1.68) | 0.17 | -0.24 | 0.16 |
| OTM – 0.8 | 1.45 | (0.06) | 0.01 | -0.01 | 0.01 |
| OTM – 0.9 | 24.86 | (1.52) | 0.15 | -0.17 | 0.16 |
| ATM – 1 | 14.04 | (1.31) | 0.13 | -0.16 | 0.13 |
| OTM – 1.1 | -26.94 | (-0.97) | -0.10 | -0.29 | -0.10 |
| OTM – 1.2 | -25.28 | (-0.75) | -0.08 | -0.26 | -0.07 |
| ATM Straddle | -4.45 | (-0.44) | -0.04 | -0.10 | -0.04 |
| ATM – 1 (DH) | -15.10 | (-1.02) | -0.10 | -0.19 | -0.10 |
| ATM Straddle (DH) | -9.87 | (-1.85) | -0.19 | -0.25 | -0.18 |

**Table 3.** Corridor variance-of-variance risk premia.

This table reports the return statistics for the variance-of-variance risk premium $\mathrm{VVP}_{B_d,B_u}$ for different corridors $B_d$ and $B_u$ over the sample period from April 2006 until August 2014. The risk premium is measured as the monthly return defined by $RV_{t,T}^{\Pi,B_d,B_u}/IV_{t,T}^{B_d,B_u} - 1$, where $B_d$ and $B_u$ are specified as described in Section **??**. Columns (4) to (6) report the Sharpe ratio, the Sortino ratio and the Stutzer index.

| Strategy | mean return | ($t$-stat) | Sharpe | Sortino Ratio | Stutzer |
|---|---|---|---|---|---|
| $\mathrm{VVP}_{0,50}$ | -15.00 | (-2.11) | -0.21 | -0.31 | -0.20 |
| $\mathrm{VVP}_{50,100}$ | -30.73 | (-2.58) | -0.26 | -0.41 | -0.24 |
| | | | | | |
| $\mathrm{VVP}_{0,33}$ | -21.64 | (-2.64) | -0.26 | -0.35 | -0.25 |
| $\mathrm{VVP}_{33,67}$ | -14.95 | (-2.06) | -0.21 | -0.31 | -0.20 |
| $\mathrm{VVP}_{67,100}$ | -37.93 | (-2.41) | -0.24 | -0.44 | -0.22 |

**Table 4.** Alphas and risk factor sensitivities.

This table reports the estimation results for the regression model $r_{it} = \alpha_i + \beta_{m,i}\,(r_t^m - r_{ft}) + \beta_{smb,i}\,r_t^{smb} + \beta_{hml,i}\,r_t^{hml} + \beta_{umd,i}\,r_t^{umd} + \beta_{vixf,i}\,r_t^{vixf} + \varepsilon_{it}$ where $r_{it}$ is the monthly excess return of VIX option strategies listed in column (1). Columns (2) to (7) report the intercept (alpha, $\alpha_i$) and the coefficients for the following risk factors: the market risk premium (MktRP, $\beta_{m,i}$), the size factor (SMB, $\beta_{smb,i}$), the book-to-market factor (HML, $\beta_{hml,i}$), the momentum factor (UMD, $\beta_{umd,i}$) and the return of VIX Futures (VIXF, $\beta_{vixf,i}$). Column (8) reports the $R^2$ of the regression. The variance-of-variance risk premium trade is labeled VVP. The sample period is from April 2006 until August 2014.

| Strategy | alpha | MktRP | SMB | HML | UMD | VIXF | $R^2$ |
|---|---|---|---|---|---|---|---|
| VVP | -17.78 | -7.27 | | | | | 0.29 |
| | (-2.98) | (-5.08) | | | | | |
| | -16.98 | -0.22 | -1.17 | -2.46 | -1.26 | 1.75 | 0.45 |
| | (-3.28) | (-0.16) | (-0.50) | (-2.02) | (-1.35) | (5.25) | |
| $\text{VVP}_{0,50}$ | -16.65 | 2.58 | | | | | 0.03 |
| | (-2.17) | (2.65) | | | | | |
| | -17.63 | -0.66 | 2.49 | -1.48 | 0.79 | -0.81 | 0.09 |
| | (-2.39) | (-0.27) | (0.59) | (-0.76) | (0.75) | (-2.30) | |
| $\text{VVP}_{50,100}$ | -21.30 | -14.71 | | | | | 0.36 |
| | (-2.54) | (-6.40) | | | | | |
| | -17.15 | -0.96 | -2.33 | -3.31 | -2.54 | 3.33 | 0.59 |
| | (-2.49) | (-0.49) | (-0.68) | (-1.28) | (-2.31) | (7.08) | |
| $\text{VVP}_{0,33}$ | -25.02 | 5.27 | | | | | 0.10 |
| | (-2.96) | (3.76) | | | | | |
| | -26.27 | 1.16 | 3.24 | -3.57 | 0.53 | -1.05 | 0.18 |
| | (-3.18) | (0.42) | (0.82) | (-1.49) | (0.52) | (-2.15) | |
| $\text{VVP}_{33,67}$ | -12.75 | -3.43 | | | | | 0.05 |
| | (-1.78) | (-2.83) | | | | | |
| | -12.74 | -4.59 | 0.91 | 0.78 | -0.37 | -0.16 | 0.06 |
| | (-1.81) | (-1.99) | (0.30) | (0.37) | (-0.26) | (-0.36) | |
| $\text{VVP}_{67,100}$ | -25.57 | -19.27 | | | | | 0.36 |
| | (-2.49) | (-5.73) | | | | | |
| | -19.86 | -0.08 | -3.68 | -2.72 | -2.87 | 4.64 | 0.60 |
| | (-2.59) | (-0.03) | (-0.80) | (-0.78) | (-1.54) | (8.22) | |

**Table 5.** Variance-of-variance risk premium (monthly term structure).

This table reports the average variance-of-variance risk premia defined as $RV^{\Pi}_{t,T}/IV_{t,T} - 1$, where $IV_{t,T}$ is the implied variance and $RV^{\Pi}_{t,T}$ the realized daily variance. Risk premia are reported for a holding period of two, three and four months. Every month the day after the VIX option expiry is selected and variance-of-variance contract returns are calculated from VIX options and futures with two, three and four months to expiry. The table also report results for corridor variance contracts $\mathrm{VVP}_{B_d,B_u}$ where $B_d$ and $B_u$ denote the lower and upper bound of the corridor.

| | 2 months | | 3 months | | 4 months | |
|---|---|---|---|---|---|---|
| | mean return | ($t$-stat) | mean return | ($t$-stat) | mean return | ($t$-stat) |
| VVP | -20.44 | (-3.25) | -23.01 | (-4.48) | -21.45 | (-4.12) |
| $\mathrm{VVP}_{0,50}$ | -6.04 | (-0.84) | -8.08 | (-1.19) | -11.13 | (-1.70) |
| $\mathrm{VVP}_{50,100}$ | -31.60 | (-2.73) | -33.98 | (-3.23) | -28.94 | (-2.56) |
| $\mathrm{VVP}_{0,33}$ | 1.45 | (0.13) | 1.91 | (0.16) | 3.30 | (0.31) |
| $\mathrm{VVP}_{33,67}$ | -19.89 | (-3.30) | -22.99 | (-3.76) | -32.68 | (-5.95) |
| $\mathrm{VVP}_{67,100}$ | -32.78 | (-1.91) | -35.46 | (-2.31) | -23.06 | (-1.38) |

**Table 6.** Alphas and risk factor sensitivities (term structure).

This table reports the estimation results for the regression model $r_{it} = \alpha_i + \beta_{m,i}\,(r_t^m - r_{ft}) + \beta_{smb,i}\,r_t^{smb} + \beta_{hml,i}\,r_t^{hml} + \beta_{umd,i}\,r_t^{umd} + \beta_{vixf,i}\,r_t^{vixf} + \varepsilon_{it}$ where $r_{it}$ is the excess return for the variance-of-variance contracts in held over different holding periods from two to four months. Columns (2) to (7) report the alpha and beta coefficients for the following risk factors: alpha ($\alpha_i$), the market risk premium (MktRP, $\beta_{m,i}$), the size factor (SMB, $\beta_{smb,i}$), the book-to-market factor (HML, $\beta_{hml,i}$), the momentum factor (UMD, $\beta_{umd,i}$) and the return of VIX Futures (VIXF, $\beta_{vixf,i}$). Column (8) reports the $R^2$ of the regression. Panel A reports results for a holding period of two months, Panel B for four-months holding period returns. All $t$-statistics are adjusted for heteroskedasticity and autocorrelation.

| Strategy | alpha | MktRP | SMB | HML | UMD | VIXF | $R^2$ |
|---|---|---|---|---|---|---|---|
| Panel A: Two-month investments | | | | | | | |
| VVP | -13.56 | -5.06 | | | | | 0.33 |
| | (-2.32) | (-6.05) | | | | | |
| | -11.86 | -0.15 | -0.27 | -0.03 | -0.67 | 1.32 | 0.51 |
| | (-2.27) | (-0.13) | (-0.14) | (-0.03) | (-1.12) | (5.37) | |
| Panel B: Three-month investments | | | | | | | |
| VVP | -14.86 | -3.50 | | | | | 0.39 |
| | (-2.72) | (-5.57) | | | | | |
| | -15.97 | 0.39 | 0.49 | -0.28 | 0.32 | 1.07 | 0.58 |
| | (-4.03) | (0.51) | (0.55) | (-0.45) | (1.14) | (6.23) | |
| Panel C: Four-month investments | | | | | | | |
| VVP | -11.95 | -3.19 | | | | | 0.45 |
| | (-2.08) | (-6.72) | | | | | |
| | -14.04 | 0.01 | -0.06 | -0.02 | 0.32 | 0.92 | 0.61 |
| | (-3.17) | (0.01) | (-0.08) | (-0.04) | (1.56) | (5.81) | |

**Table 7.** Variance-of-variance risk premium (Holding Periods).

This table reports the statistics for the monthly variance-of-variance risk premium calculated from longer-term investments. $\mathrm{VVP}^{x \to y}$ denotes the premium of a contract initiated $x$ months before maturity and held for $y$ months over the sample period from April 2006 until August 2014. Columns (4) to (6) report the Sharpe ratio, the Sortino ratio and the Stutzer index.

| Strategy | mean return | ($t$-stat) | Sharpe | Sortino Ratio | Stutzer |
|---|---|---|---|---|---|
| $\mathrm{VVP}^{2\to 1}$ | -4.55 | (-1.14) | -0.11 | -0.20 | -0.11 |
| $\mathrm{VVP}^{3\to 1}$ | -3.16 | (-1.06) | -0.11 | -0.19 | -0.10 |
| $\mathrm{VVP}^{4\to 1}$ | -0.70 | (-0.29) | -0.03 | -0.05 | -0.03 |

**Table 8.** Alphas and risk factor sensitivities (term structure).

This table reports the estimation results for the regression model $r_{it} = \alpha_i + \beta_{m,i}\,(r_t^m - r_{ft}) + \beta_{smb,i}\,r_t^{smb} + \beta_{hml,i}\,r_t^{hml} + \beta_{umd,i}\,r_t^{umd} + \beta_{vixf,i}\,r_t^{vixf} + \varepsilon_{it}$ where $r_{it}$ is the excess return of different variance-of-variance contracts in column (1). VVP$^{x \to y}$ denotes the premium of a contract initiated $x$ months before maturity and held for $y$ months over the sample period from April 2006 until August 2014. Columns (2) to (7) report the alpha and beta coefficients for the following risk factors: alpha ($\alpha_i$), the market risk premium (MktRP, $\beta_{m,i}$), the size factor (SMB, $\beta_{smb,i}$), the book-to-market factor (HML, $\beta_{hml,i}$), the momentum factor (UMD, $\beta_{umd,i}$) and the return of VIX Futures (VIXF, $\beta_{vixf,i}$). Column (8) reports the $R^2$ of the regression.

| Strategy | alpha | MktRP | SMB | HML | UMD | VIXF | $R^2$ |
|---|---|---|---|---|---|---|---|
| VVP$^{2\to 1}$ | -0.49 | -4.63 | | | | | 0.36 |
| | (-0.17) | (-5.81) | | | | | |
| | 1.13 | -2.45 | 1.61 | -1.42 | -1.89 | 1.13 | 0.53 |
| | (0.38) | (-2.41) | (1.16) | (-1.44) | (-2.91) | (2.83) | |
| VVP$^{3\to 1}$ | -0.03 | -3.54 | | | | | 0.40 |
| | (-0.01) | (-7.25) | | | | | |
| | 0.39 | -2.06 | 0.57 | -1.65 | -1.27 | 0.94 | 0.54 |
| | (0.21) | (-2.79) | (0.53) | (-1.81) | (-2.58) | (3.26) | |
| VVP$^{4\to 1}$ | 1.38 | -2.54 | | | | | 0.36 |
| | (0.79) | (-6.69) | | | | | |
| | 1.47 | -1.38 | 0.98 | -1.59 | -0.82 | 0.85 | 0.50 |
| | (0.86) | (-2.57) | (1.15) | (-2.35) | (-2.18) | (3.22) | |

**Table 9.** Variance risk premium.

This table reports the return statistics for the variance risk premium (VP) calculated from S&P 500 index options from April 2006 until August 2014. VP is measured as the monthly return defined by $RV_{t,T}^{\Pi}/IV_{t,T} - 1$, where $IV_{t,T}$ is the implied variance and $RV_{t,T}^{\Pi}$ the realized daily variance of S&P 500 index futures over a monthly period. Columns (4) to (6) report the Sharpe ratio, the Sortino ratio and the Stutzer index.

| Strategy | mean return | ($t$-stat) | Sharpe | Sortino Ratio | Stutzer |
|---|---|---|---|---|---|
| Panel A: January 1996 to August 2014 | | | | | |
| VP | -20.77 | (-4.70) | -0.31 | -0.48 | -0.27 |
| Panel B: April 2006 to August 2014 | | | | | |
| VP | -16.68 | (-2.03) | -0.20 | -0.38 | -0.19 |

**Table 10.** Alphas and risk factor sensitivities (variance risk premium).

This table reports the estimation results for the regression model $r_{it} = \alpha_i + \beta_{m,i}(r_t^m - r_{ft}) + \beta_{smb,i} r_t^{smb} + \beta_{hml,i} r_t^{hml} + \beta_{umd,i} r_t^{umd} + \varepsilon_{it}$ where $r_{it}$ is the return of the variance contract in column (1). VP is the standard variance contract. Columns (2) to (6) report the alpha and beta coefficients for the following risk factors: alpha ($\alpha_i$), the market risk premium (MktRP, $\beta_{m,i}$), the size factor (SMB, $\beta_{smb,i}$), the book-to-market factor (HML, $\beta_{hml,i}$) and the momentum factor (UMD, $\beta_{umd,i}$). Column (7) reports the $R^2$ of the regression. Panel A reports results for a holding period of two months, Panel B and C focus on three and fours months. All $t$-statistics are corrected for heteroskedasticity and autocorrelation.

| Strategy | alpha | MktRP | SMB | HML | UMD | $R^2$ |
|---|---|---|---|---|---|---|
| Panel A: January 1996 to August 2014 | | | | | | |
| VP | -16.20 | -7.23 | | | | 0.34 |
| | (-3.69) | (-5.26) | | | | |
| | -13.43 | -7.68 | -2.96 | -3.05 | -2.13 | 0.39 |
| | (-2.90) | (-5.34) | (-2.41) | (-2.05) | (-3.05) | |
| Panel B: April 2006 to August 2014 | | | | | | |
| VP | -9.78 | -10.37 | | | | 0.50 |
| | (-1.46) | (-5.68) | | | | |
| | -8.61 | -12.95 | 2.55 | 4.51 | -1.80 | 0.56 |
| | (-1.27) | (-6.06) | (1.01) | (1.47) | (-1.91) | |

**Table 11.** Variance-of-Variance contract alpha (after adjusting for variance risk).

This table reports the estimation results for the regression model $r_{it} = \alpha_i + \beta_{m,i}\,(r_t^m - r_{ft}) + \beta_{vp,i}\,r_t^{vp} + \beta_{smb,i}\,r_t^{smb} + \beta_{hml,i}\,r_t^{hml} + \beta_{umd,i}\,r_t^{umd} + \varepsilon_{it}$ where $r_{it}$ is the return of the variance-of-variance contract. Columns (2) to (7) report the alpha and beta coefficients for the following risk factors: alpha ($\alpha_i$), the market risk premium (MktRP, $\beta_{m,i}$), the return of the variance contract (VP, $\beta_{vp,i}$), the size factor (SMB, $\beta_{smb,i}$), the book-to-market factor (HML, $\beta_{hml,i}$) and the momentum factor (UMD, $\beta_{umd,i}$). Column (8) reports the $R^2$ of the regression.

| Strategy | alpha | MktRP | VP | SMB | HML | UMD | $R^2$ |
|---|---|---|---|---|---|---|---|
| VVP | -15.55 | -0.63 | 0.50 | | | | 0.60 |
| | (-3.07) | (-0.58) | (4.91) | | | | |
| | -15.53 | 0.36 | 0.53 | -0.39 | -4.23 | -0.55 | 0.61 |
| | (-3.18) | (0.24) | (5.42) | (-0.16) | (-2.52) | (-0.61) | |

**Table 12.** Variance-of-Variance Risk Premium (transaction cost).

This table reports average monthly variance-of-variance contract returns after accounting for transaction cost. Panel A reports the average P&L and corresponding $t$-statistics for a daily rebalancing interval, for strategies with a fixed notional $N(\$) = 100$ and for $N(\$) = 100 \times \left[IV_{t,T}^{\Pi,B_d,B_u}\right]^{-1}$. *Short (Mid)* is the short VVP trade based on mid-option quotes, whereas *Short (TC)* accounts for bid-ask spreads. Corridor variance swaps are calculated using the smallest and largest available strike as $B_d$ and $B_u$. Panel B reports empirical results for a weekly rebalancing scheme.

| Notional | $N(\$) = 100$ | | $N(\$) = 100 \times \left[IV_{t,T}^{\Pi,B_d,B_u}\right]^{-1}$ | |
|---|---|---|---|---|
| | Average $P\&L$ | $t$-stat | Average $P\&L$ | $t$-stat |
| Panel A: Daily Rebalancing | | | | |
| Short (Mid) | 1.70 | (3.93) | 24.16 | (3.39) |
| Short (TC) | 1.00 | (2.33) | 14.01 | (1.74) |
| Short (Mid) Corridor Swap | 1.61 | (3.73) | 23.04 | (3.18) |
| Short (TC) Corridor Swap | 0.99 | (2.31) | 13.88 | (1.71) |
| Panel B: Weekly Rebalancing | | | | |
| Short (Mid) | 2.39 | (4.80) | 35.10 | (4.44) |
| Short (TC) | 1.69 | (3.44) | 26.88 | (3.08) |
| Short (Mid) Corridor Swap | 2.30 | (4.64) | 34.17 | (4.26) |
| Short (TC) Corridor Swap | 1.68 | (3.43) | 26.76 | (3.05) |

**Table 13.** Variance and variance-of-variance risk premium in structural benchmark models.

This tables shows the distribution of the monthly variance-of-variance risk premia (Panel A) and variance risk premia (Panel B) in the various benchmark models introduced in Sections 4.1 expressed as percentage returns. The models are abbreviated as follows: SV (stochastic volatility of Heston, 1993), SVJ (stochastic volatility and jump model of Bates, 1996), SVCJ (stochastic volatility and contemporaneous jump model of Eraker *et al.*, 2003) and SVSMR/SVSMRJ (stochastic volatility, stochastic mean reversion and contemporaneous jump model of Bardgett *et al.*, 2013). One simulation run consists of 8 years of monthly returns which are averaged to obtain a simulated estimate for the VVP/VP. The mean, standard deviation and percentiles (perc.) of the distribution of average variance-of-variance/variance risk premia are based on 500 simulation runs.

| Model | SV | SVJ | SVCJ | SVSMR | SVSMRJ |
|---|---|---|---|---|---|
| Panel A: Variance-of-variance risk premium | | | | | |
| **mean** | 4.30 | 2.09 | 5.56 | 4.16 | 42.17 |
| **stdev** | 4.04 | 3.87 | 16.00 | 5.53 | 25.93 |
| **perc.** | | | | | |
| 1% | -5.04 | -5.81 | -14.77 | -9.05 | 4.15 |
| 5% | -2.34 | -4.10 | -10.61 | -4.01 | 11.70 |
| 10% | -0.71 | -2.73 | -8.07 | -2.77 | 16.56 |
| 50% | 4.19 | 1.98 | 1.14 | 4.01 | 36.88 |
| 90% | 9.75 | 7.41 | 24.28 | 11.59 | 69.39 |
| 95% | 11.09 | 8.86 | 38.95 | 13.23 | 87.70 |
| 99% | 13.10 | 12.39 | 70.01 | 17.58 | 151.53 |
| Panel B: Variance risk premium | | | | | |
| **mean** | -13.14 | -12.27 | -22.04 | 24.22 | -6.49 |
| **stdev** | 6.19 | 8.18 | 8.10 | 38.07 | 25.61 |
| **perc.** | | | | | |
| 1% | -23.69 | -27.38 | -35.87 | -23.04 | -37.79 |
| 5% | -21.14 | -23.74 | -33.06 | -16.08 | -31.33 |
| 10% | -19.42 | -21.60 | -31.40 | -11.28 | -27.61 |
| 50% | -13.19 | -13.42 | -23.13 | 14.40 | -11.92 |
| 90% | -7.17 | -2.67 | -11.33 | 71.99 | 17.20 |
| 95% | -5.27 | 2.57 | -7.39 | 92.04 | 41.81 |
| 99% | -1.41 | 12.55 | -0.14 | 165.86 | 87.02 |

**Table 14.** Variance-of-variance risk premium in extended benchmark models.

This tables shows the distribution of variance-of-variance risk premia in the various benchmark models introduced in Section 4.2 expressed as percentage returns. The models are abbreviated as follows: CTOUSV (reduced-form model of Mencía and Sentana, 2013), M1 and M2 are based on $d\omega_t = \kappa_\omega^{\mathbb{Q}} \left(\theta_\omega^{\mathbb{Q}} - \omega_t\right) dt + \sigma_\omega \sqrt{\omega_t}\, dW_t^{\omega,\mathbb{Q}}$ and $dW_t^{\omega,\mathbb{Q}} = dW_t^{\omega,\mathbb{P}} + \eta_\omega \sqrt{\omega_t} dt$ where M1 assumes a negative risk premium $\eta_\omega$ and in M2 the risk premium is set to zero. The suffix S1 for CTOUSV indicates that parameters for the shorter sample from Mencía and Sentana (2013) are used whereas S2 indicates parameter estimates from the whole sample period. One simulation run consists of 8 years of monthly variance-of-variance risk premium returns which are averaged to obtain a simulated estimate for the variance-of-variance risk premium. The mean, standard deviation and percentiles (perc.) of the distribution of average variance-of-variance risk premia are based on 500 simulation runs.

| Model | CTOUSV-S1 | CTOUSV-S2 | M1 | M2 |
|---|---|---|---|---|
| **mean** | 3.29 | 2.73 | -28.54 | -2.15 |
| **stdev** | 17.23 | 14.13 | 3.87 | 3.54 |
| **percentile** | | | | |
| 1% | -35.78 | -32.49 | -36.56 | -9.60 |
| 5% | -18.46 | -17.59 | -34.52 | -8.21 |
| 10% | -11.99 | -12.83 | -33.07 | -6.77 |
| 50% | 2.61 | 1.86 | -28.92 | -2.01 |
| 90% | 17.99 | 17.67 | -23.27 | 2.21 |
| 95% | 24.51 | 25.65 | -21.87 | 3.84 |
| 99% | 49.65 | 44.33 | -18.44 | 5.75 |

**Table 15.** Variance and variance-of-variance risk premium in self-exciting benchmark models.

This tables shows the distribution of the monthly variance-of-variance risk premia (Panel A) and variance risk premia (Panel B), expressed as percentage returns. The models are abbreviated as follows: Model 1 refers to a self-exciting jump model with time-varying mean-reversion level and additional price jumps, Model 2 provides results for a specification with constant mean-reversion level and additional price jumps. Model 3 restricts the average size of risk-neutral and real-world variance jumps to the same value and Model 4 imposes a no-price jump restriction. Detailed parameter values are provided in the text. One simulation run consists of 8 years of monthly returns which are averaged to obtain a simulated estimate for the VVP/VP. The mean, standard deviation and percentiles (perc.) of the distribution of average variance-of-variance/variance risk premia are based on 500 simulation runs.

| Model | Model 1 | Model 2 | Model 3 | Model 4 |
|---|---|---|---|---|
| Panel A: One-month variance-of-variance risk premia | | | | |
| **mean** | -28.93 | -28.82 | 7.82 | -29.93 |
| **stdev** | 4.58 | 4.45 | 7.00 | 4.53 |
| **percentile** | | | | |
| 1% | -39.42 | -39.54 | -6.07 | -41.16 |
| 5% | -36.63 | -35.84 | -2.34 | -37.24 |
| 10% | -35.05 | -34.25 | -0.33 | -35.81 |
| 50% | -28.78 | -28.99 | 7.39 | -30.01 |
| 90% | -23.39 | -23.06 | 16.85 | -24.43 |
| 95% | -20.97 | -21.17 | 21.16 | -22.25 |
| 99% | -18.17 | -17.56 | 27.71 | -19.37 |
| Panel B: One-month variance risk premia | | | | |
| **mean** | -26.35 | -26.03 | -13.86 | -22.84 |
| **stdev** | 5.90 | 5.59 | 5.60 | 5.55 |
| **percentile** | | | | |
| 1% | -40.32 | -39.99 | -27.44 | -36.49 |
| 5% | -36.49 | -35.47 | -23.07 | -32.21 |
| 10% | -33.92 | -33.41 | -21.29 | -29.96 |
| 50% | -26.44 | -26.30 | -13.95 | -22.90 |
| 90% | -18.74 | -18.75 | -6.76 | -15.79 |
| 95% | -16.56 | -16.71 | -4.43 | -13.72 |
| 99% | -12.67 | -13.58 | -0.34 | -11.21 |

**Figure 1.** VVIX indices.

This figure shows the evolution of VVIX indices for three different constant maturities (45, 90 and 135 days). The indices measure the variance of VIX Futures implied in the VIX option prices and are constructed according to $(T-t)^{-1}\, IV_{t,T}$ where $IV_{t,T}$ is defined in Equation (2).

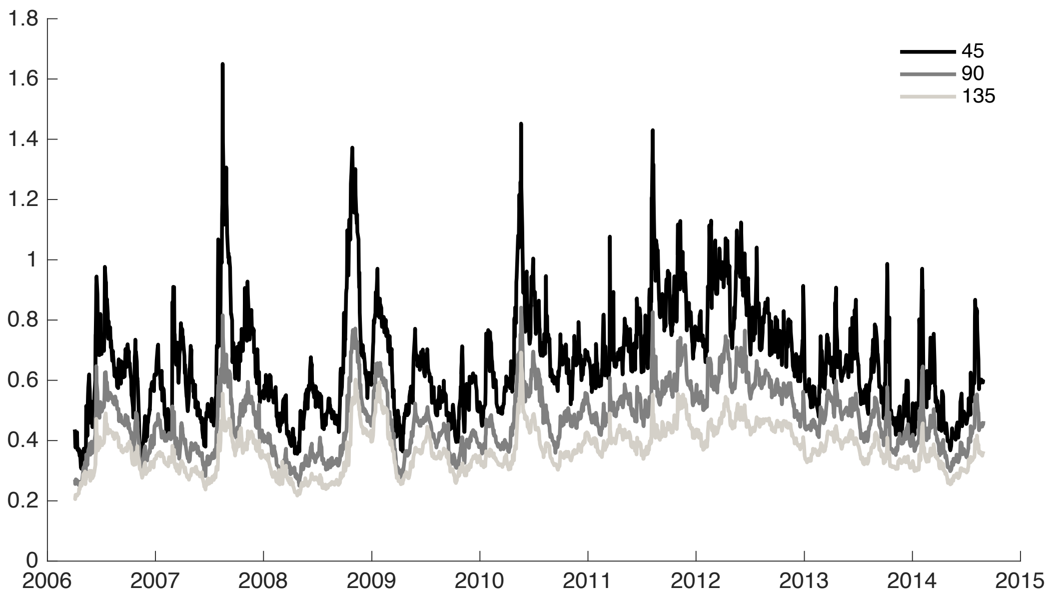

**Figure 2.** Variance-of-variance risk premia.

The left graph shows the monthly realized variances $RV^{\Pi}_{t,T}$ of VIX futures in the last month before their expiry (defined in Equation (1)) and the implied characteristic $IV_{t,T}$ constructed from VIX options as in Equation (2). The right graph provides the corresponding (excess) returns which are defined as $RV^{\Pi}_{t,T}/IV_{t,T} - 1$. Each month the return is calculated such that $T$ is the expiry date of the VIX options.

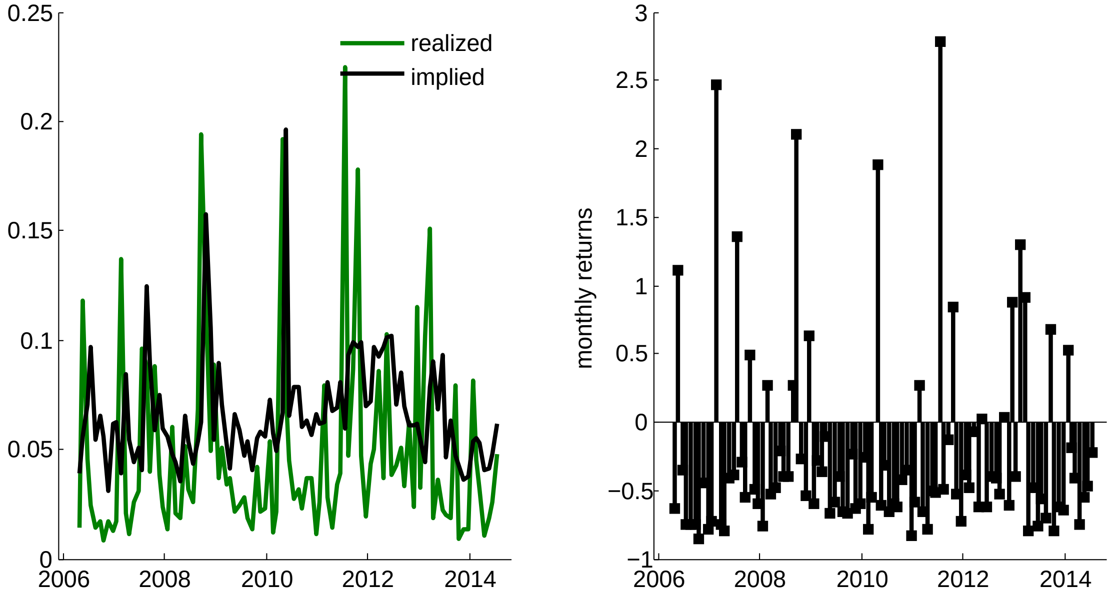

**Figure 3.** Realized vs implied characteristics for corridor variance-of-variance swaps.

The top left graph shows the monthly realized up-variances with barriers $B_d = F_{t_0,T}$ and $B_u = \infty$ and the corresponding implied characteristics. The implied characteristic constructed from VIX options are calculated using Equation (3). The top right graph provides the corresponding (excess) returns which are defined as $RV_{t,T}^{\Pi,B_d,B_u}/IV_{t,T}^{B_d,B_u} - 1$. Each month the return is calculated such that $T$ is the expiry date of VIX options. The bottom left and right graphs correspond to the down-variance with $B_d = 0$ and $B_u = F_{t_0,T}$.

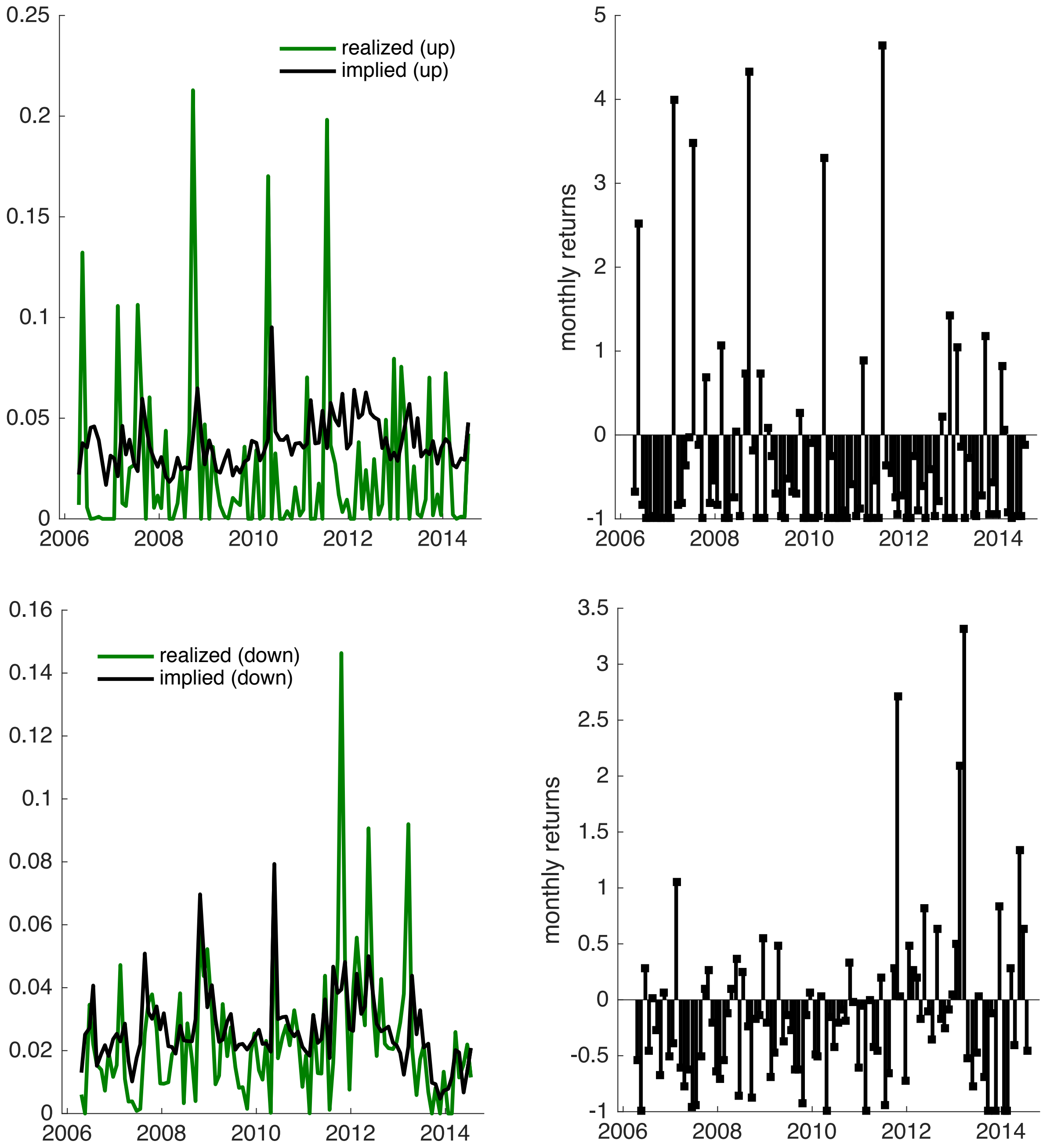

**Figure 4.** Correlation between realized variance measures (S&P 500, VIX).

The left graph provides a histogram of the correlation between monthly realized variance of the S&P 500 and VIX index for the SV model. The histogram is based on 100,000 simulated eight year samples. Parameters for the simulation are from Eraker (2004). The right histogram corresponds to the CEV model with $\gamma = 1$.